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Eigenmode orthogonality breaking and anomalous dynamics in multimode nano-optomechanical systems under non-reciprocal coupling

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Thermal motion of nanomechanical probes directly impacts their sensitivities to external forces. Its proper understanding is therefore critical for ultimate force sensing. Here, we investigate a vectorial force field sensor: a singly-clamped nanowire oscillating along two quasi-frequency-degenerate transverse directions. Its insertion in a rotational optical force field couples its eigenmodes non-symmetrically, causing dramatic modifications of its mechanical properties. In particular, the eigenmodes lose their intrinsic orthogonality. We show that this circumstance is at the origin of an **anomalous excess of noise** and of a violation of the fluctuation dissipation relation. Our model, which quantitatively accounts for all observations, provides a novel modified version of the fluctuation dissipation theorem that remains valid in non-conservative rotational force fields, and that reveals the prominent role of non-axial mechanical susceptibilities. These findings help understand the intriguing properties of thermal fluctuations in non-reciprocally-coupled multimode systems.

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Journal Club 2018-09-20

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Biography

2006: PhD, Laboratoire Kastler Brossel(Advisor Professor Antonine Heidmann)
2007-2009: Postdoctoral Scholar, Laboratory of Photonics and Quantum measurements(École polytechnique fédérale de Lausanne)

Institut Néel, Grenoble, France (permanent researcher)

Research interest

Optomechanics

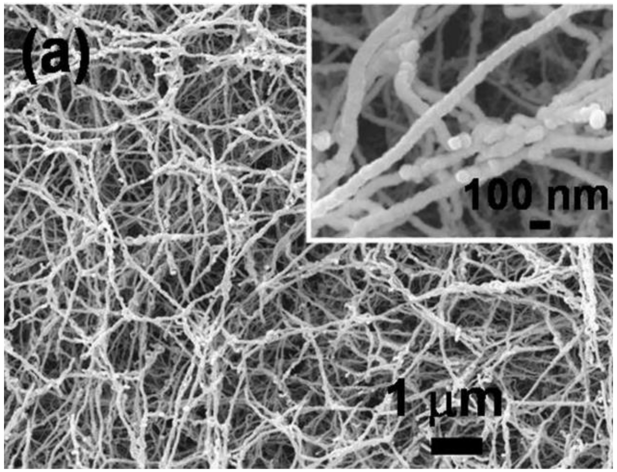
Quantum to classical conversion

Nano-Optics and forces

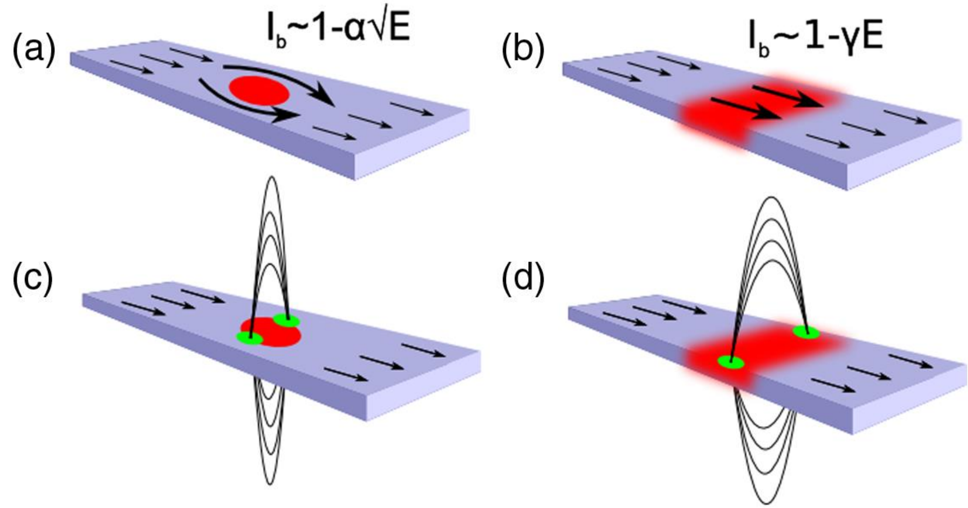
http://neel.cnrs.fr/spip.php?article260&personne=olivier.arcizet

Nanowire

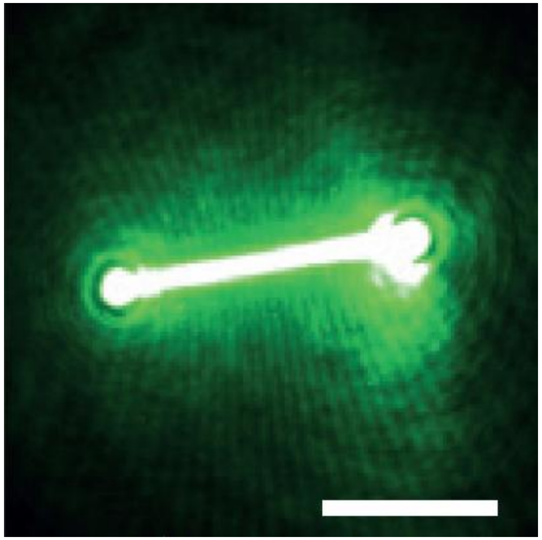
Nanowire



Sci Rep **5**, 11040 (2015).



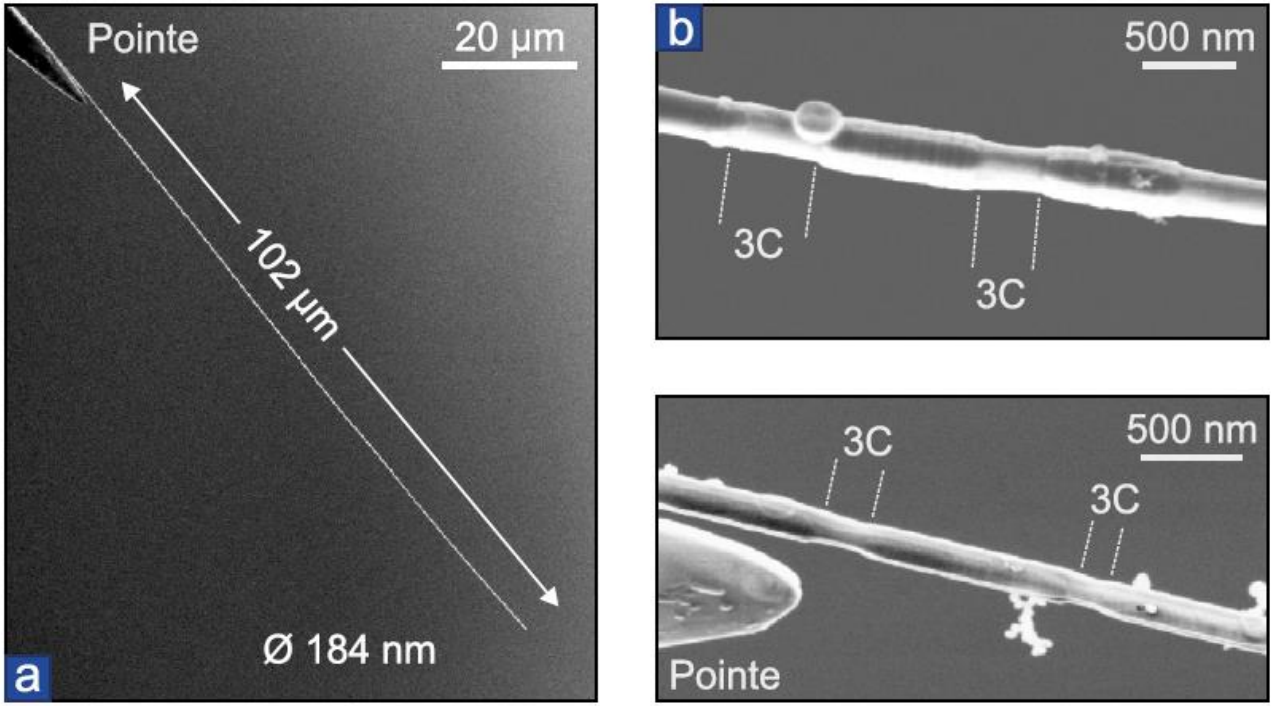
PRL **112**, 117604 (2014).



Nat. Mater. **14**, 636 (2015).

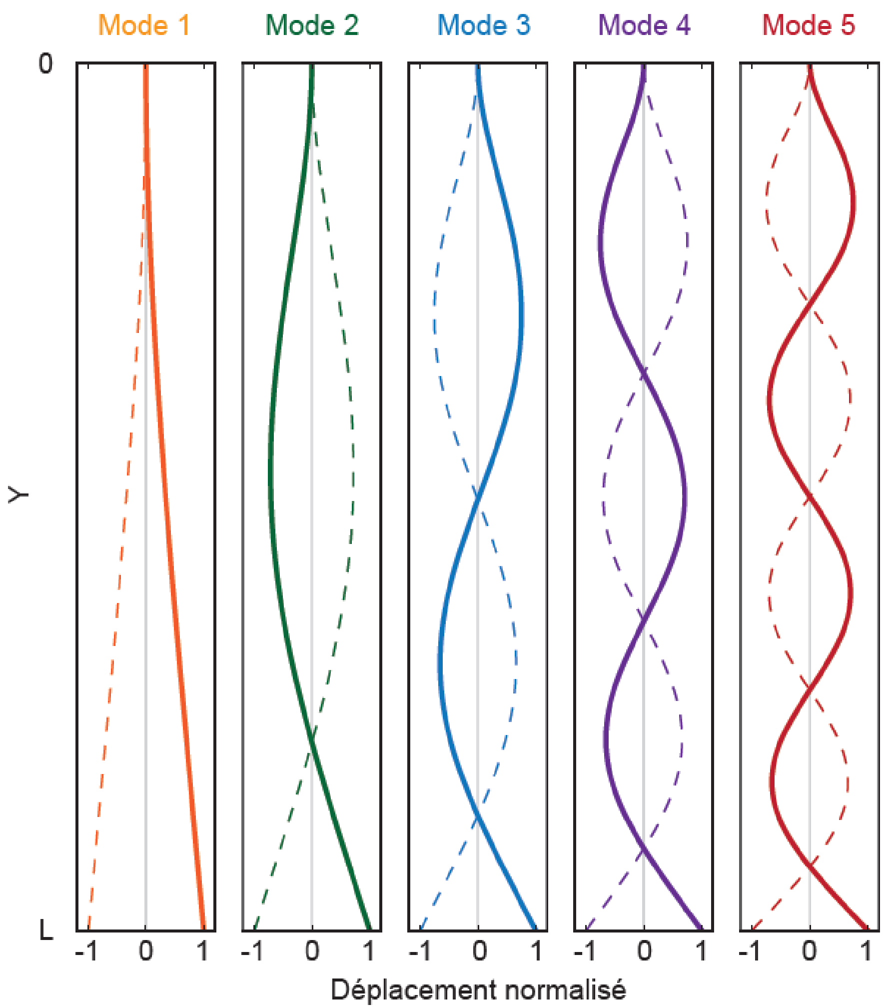
- **Molecule detection**
- **Single photon detector**
- **Nanowire laser**

Nanowire

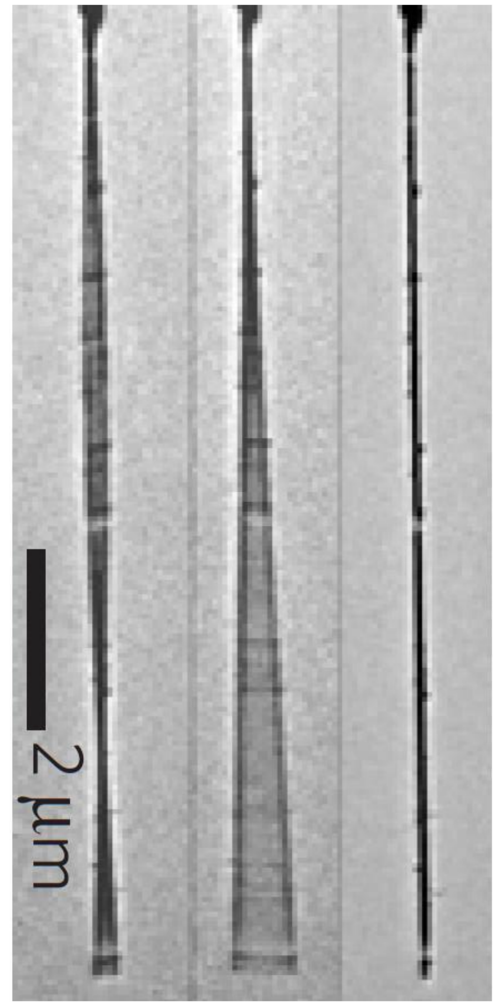


M. Bechelany, These de doctorat (2006).

Electrostatic excitation

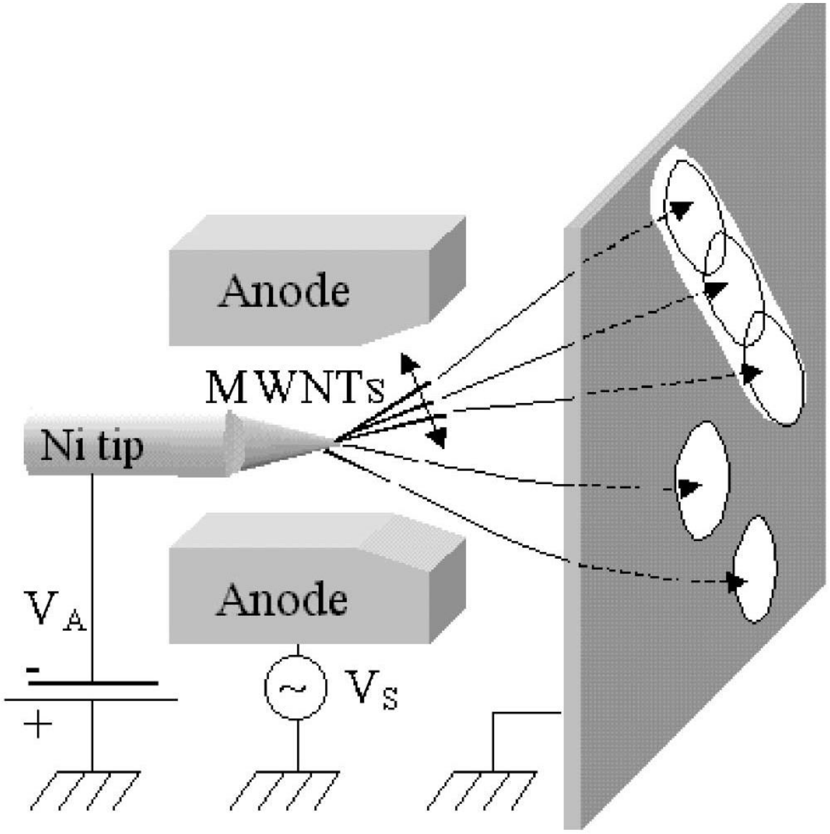


A. Gloppe, These de doctorat (2014).

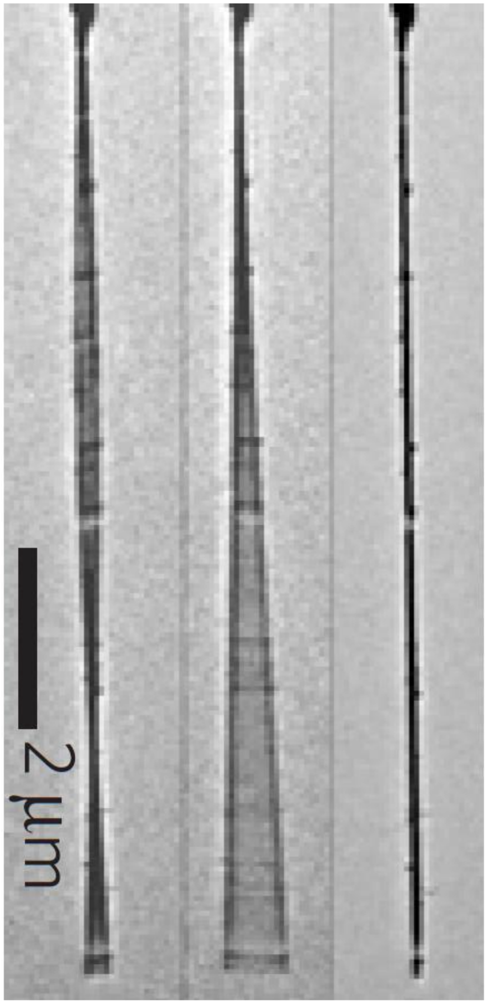


Nature Phys. **7**, 879 (2011).

Electrostatic excitation



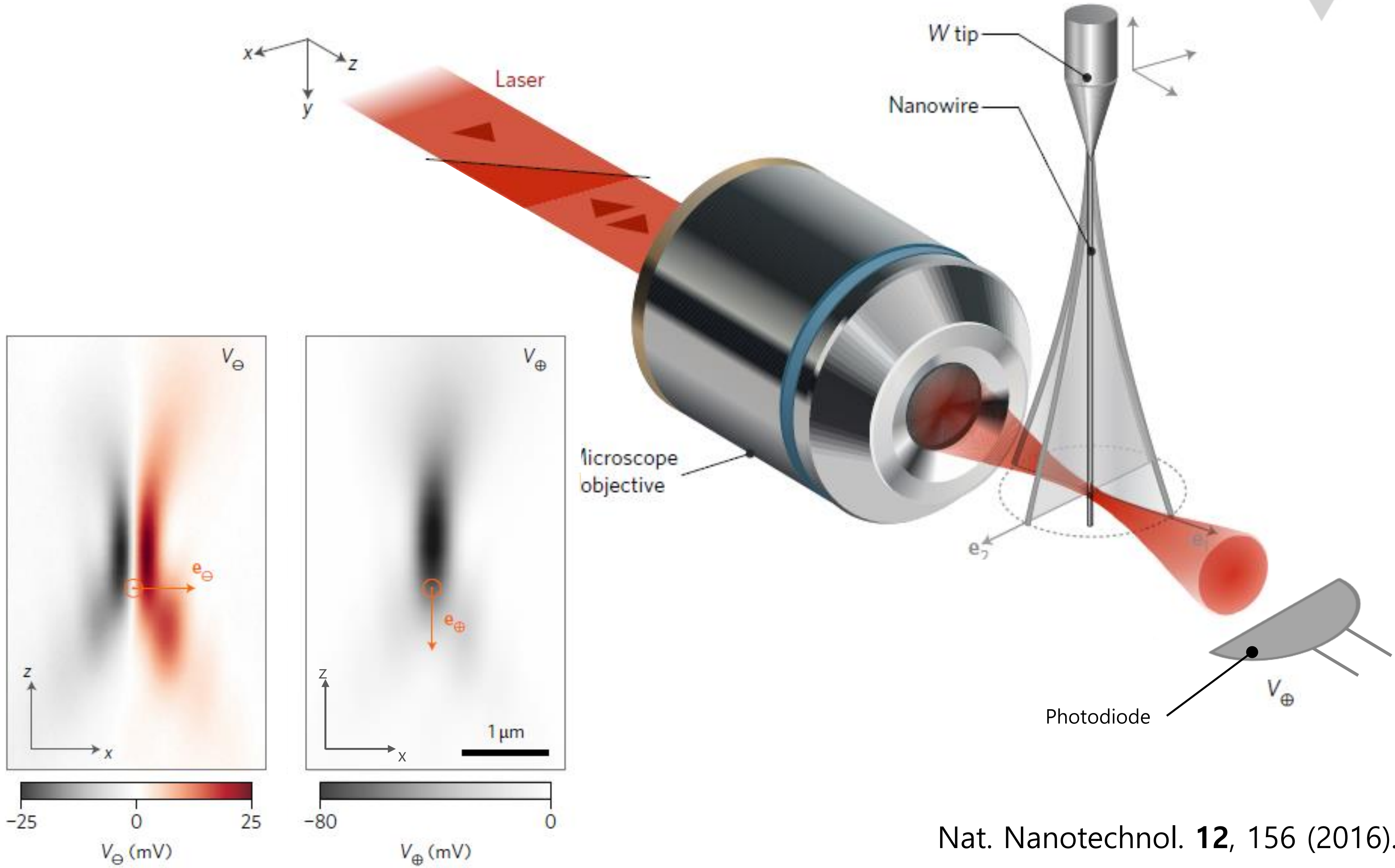
PRL, **89**, 276103 (2002).



Nature Phys. **7**, 879 (2011).

Ultrasensitive displacement measurement

Displacement detection



Nat. Nanotechnol. **12**, 156 (2016).

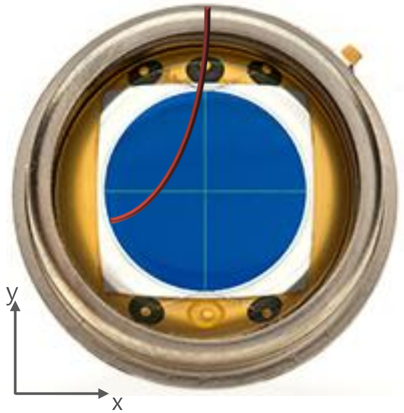
Displacement detection

$$V_{\ominus,\oplus}(\boldsymbol{r_0} + \delta\boldsymbol{r}(t)) \simeq V_{\ominus,\oplus}(\boldsymbol{r_0}) + (\delta\boldsymbol{r}(t) \cdot \boldsymbol{\nabla})\mathrm{V}_{\ominus,\oplus}|_{\mathrm{r}_0}$$

$$\delta V_{\ominus,\oplus}(t) = \boldsymbol{\delta r}(t) \cdot \boldsymbol{\nabla} V_{\ominus,\oplus}$$

$$\boldsymbol{\delta r}(t) \cdot \boldsymbol{e}_{\ominus,\oplus} = \delta V_{\ominus,\oplus}(t) \,/\, \left|\boldsymbol{\nabla} V_{\ominus,\oplus}\right|$$

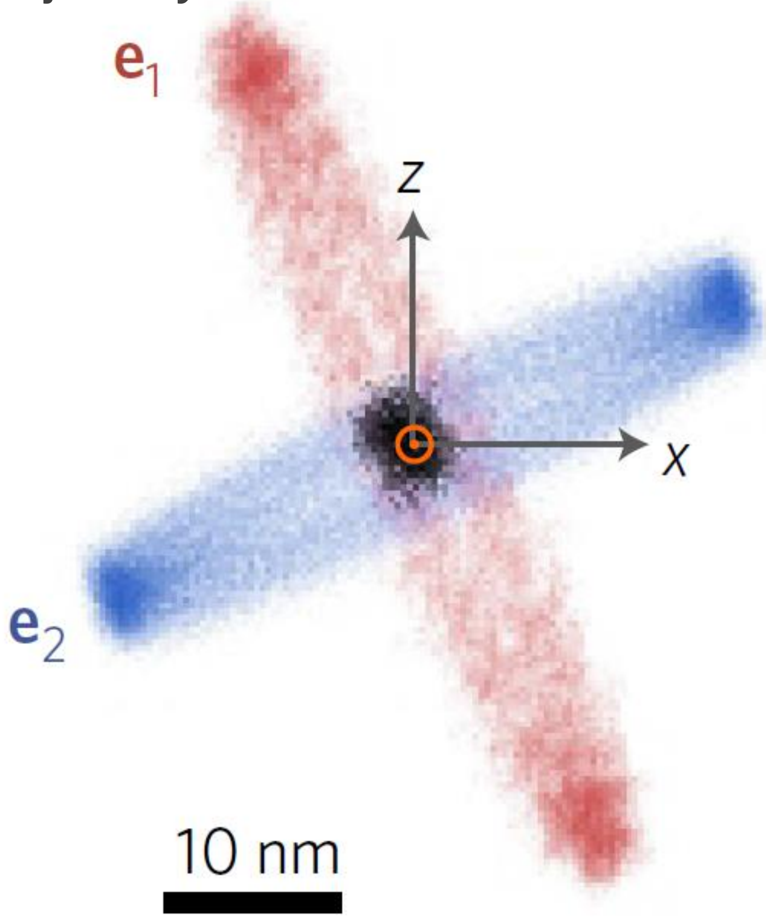
$$\boldsymbol{e}_{\ominus,\oplus} = \boldsymbol{\nabla} V_{\ominus,\oplus} \,/\, \left|\boldsymbol{\nabla} V_{\ominus,\oplus}\right|$$

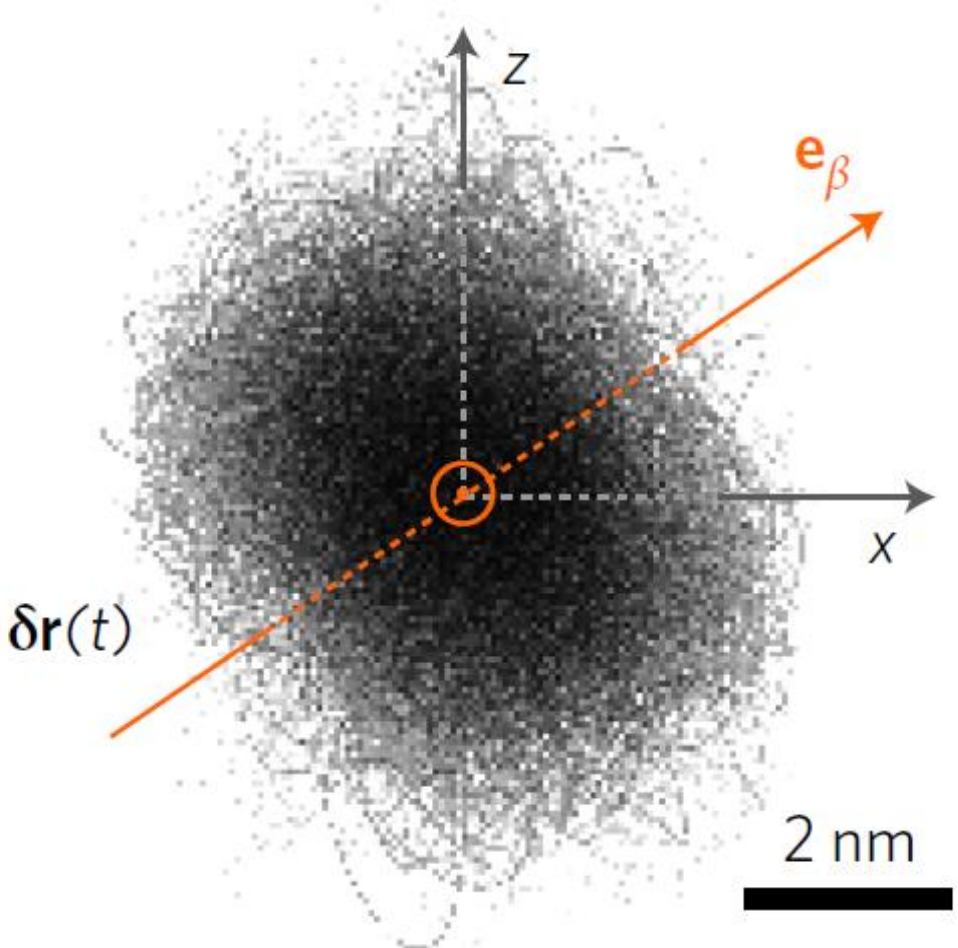


Quadrant Photodiode

Brownian motion

trajectory of nanowire

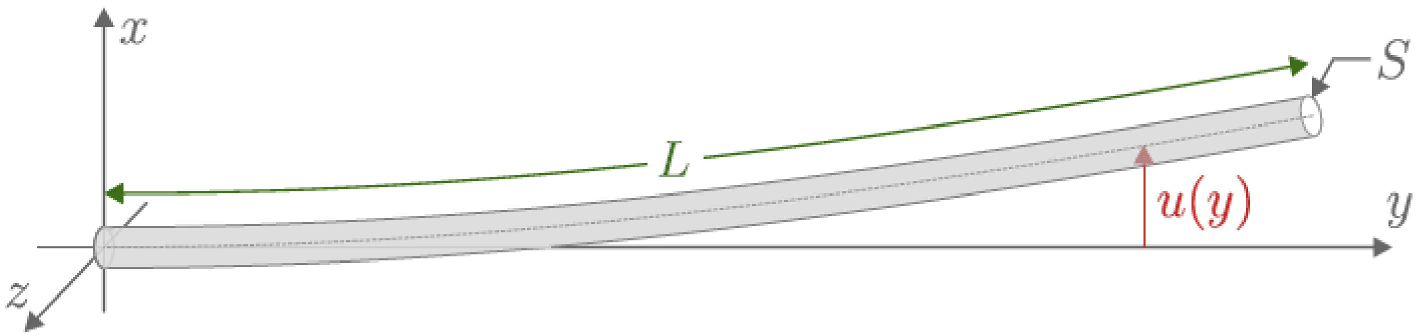




Nat. Nanotechnol. **12**, 156 (2016).

Nat. Nanotechnol. **9**, 920 (2014).

Damped harmonic oscillator



Euler–Bernoulli equation

$$\rho S \frac{\partial^2 u}{\partial t^2}(y,t) + EI \frac{\partial^4 u}{\partial y^4}(y,t) = 0$$

Mercier de Lepinay, These de doctorat (2017).

Damped harmonic oscillator

$$\delta\ddot{r} = -\Omega^2\delta r - \Gamma\delta\dot{r} + F\,/\,M_{eff}$$

$$\delta\ddot{\boldsymbol{r}} = -\begin{pmatrix}\Omega_1^2 & 0\\ 0 & \Omega_2^2\end{pmatrix}\delta\boldsymbol{r} - \begin{pmatrix}\Gamma_1 & 0\\ 0 & \Gamma_2\end{pmatrix}\delta\dot{\boldsymbol{r}} + \delta\boldsymbol{F_{th}}\,/\,M_{eff}$$

Mechanical susceptibility($\boldsymbol{\chi}$)

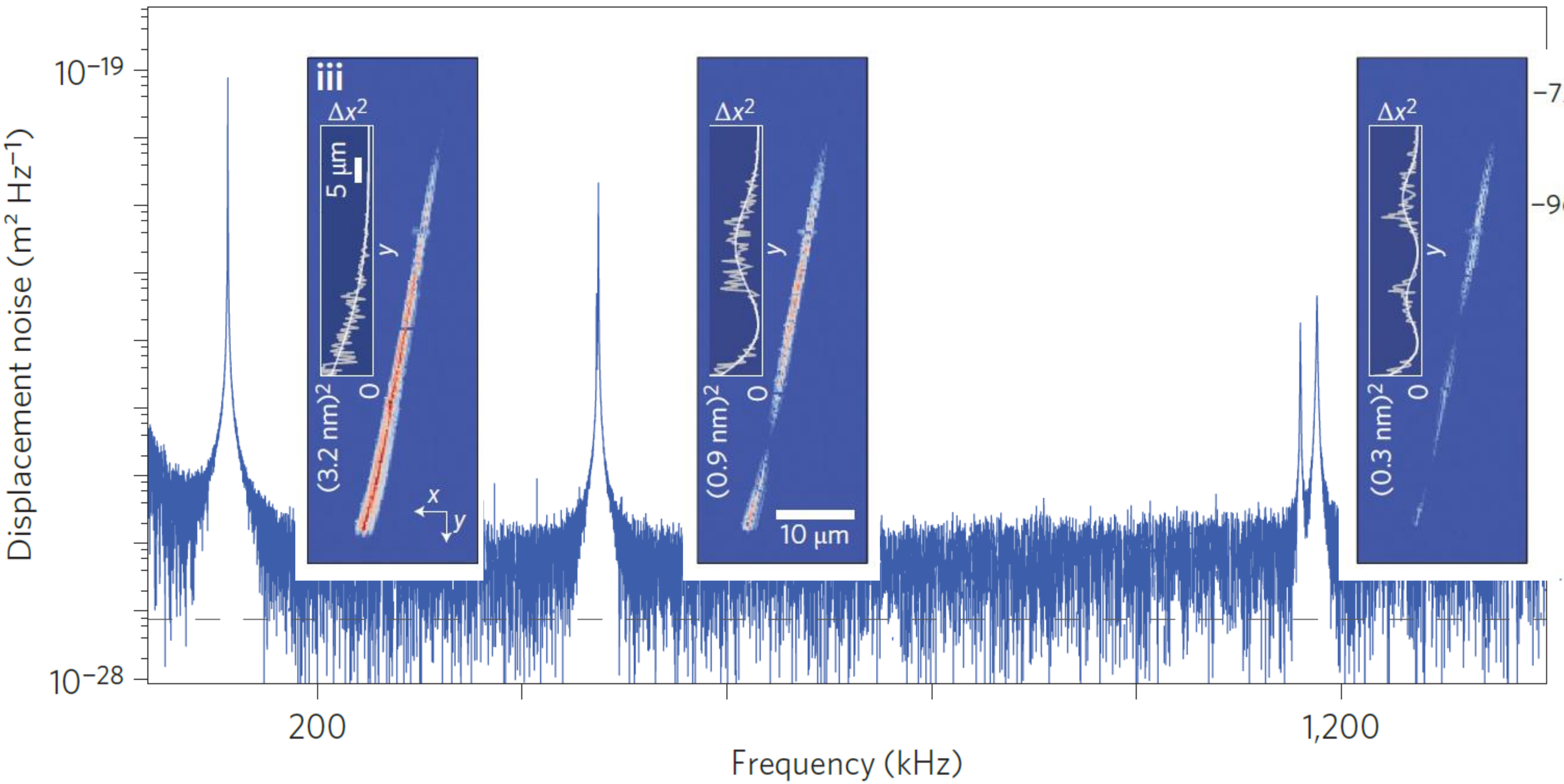
$$\delta\boldsymbol{r}[\Omega] \equiv \chi[\Omega]\delta\boldsymbol{F_{th}}[\Omega] \qquad \chi^{-1}[\Omega] = M_{eff}\begin{pmatrix}\Omega_1^2 - \Omega^2 - i\Gamma_1\Omega & 0\\ 0 & \Omega_2^2 - \Omega^2 - i\Gamma_2\Omega\end{pmatrix}$$

$$\delta\boldsymbol{r}[\Omega] \equiv \int \delta\boldsymbol{r}(t)e^{-i\Omega t}dt$$

Thermal noise spectrum

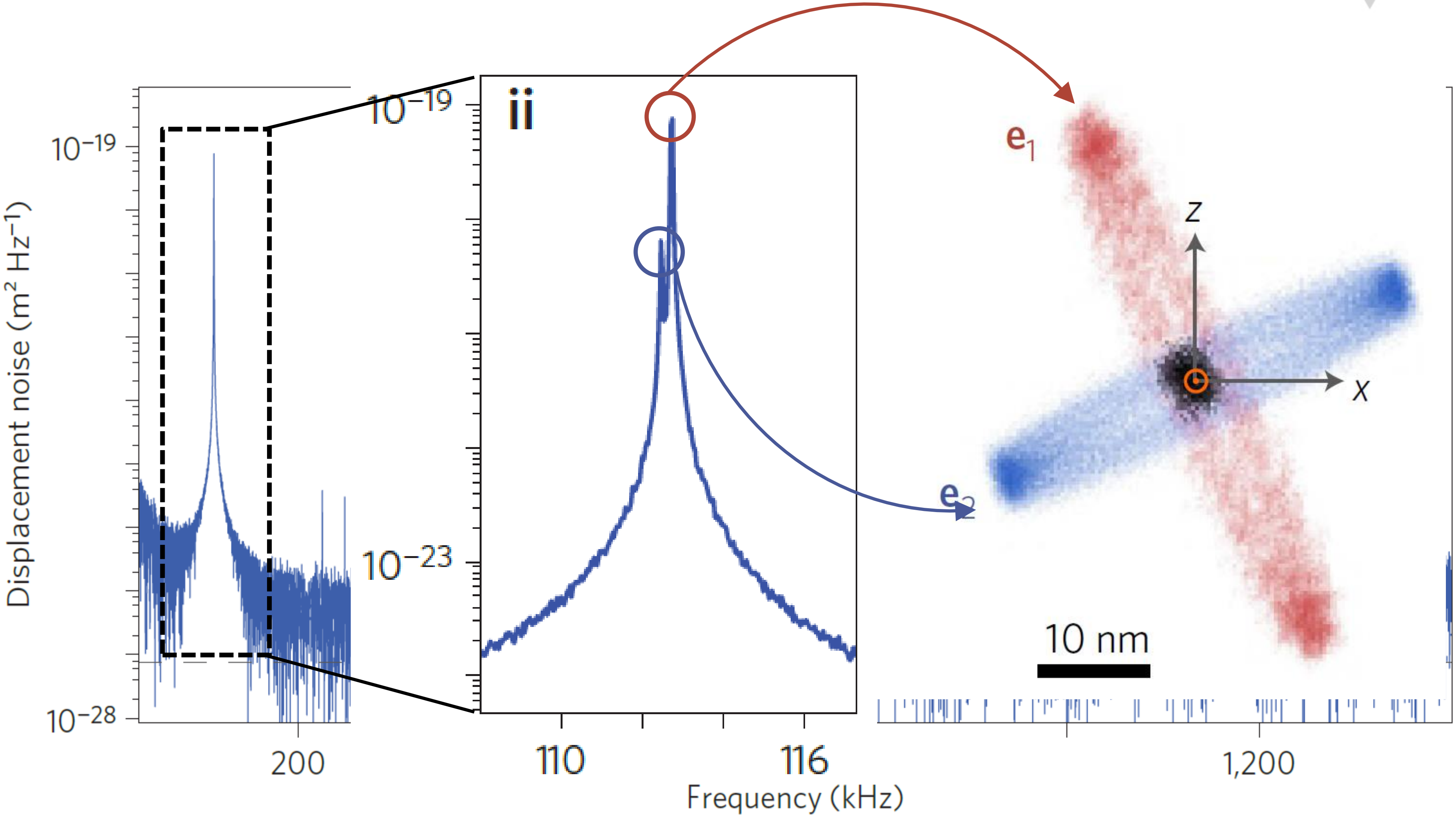
$$S_{\delta r_1}[\Omega] = \langle\delta r_1[\Omega]\delta r_1^*[\Omega]\rangle = |\chi_{11}|^2 S_{F_{th,1}} \qquad S_{F_{th,1}} = 2M_{eff}\Gamma_1 k_B T$$

Power spectrum of displacement



Nat. Nanotechnol. **9**, 920(2014).

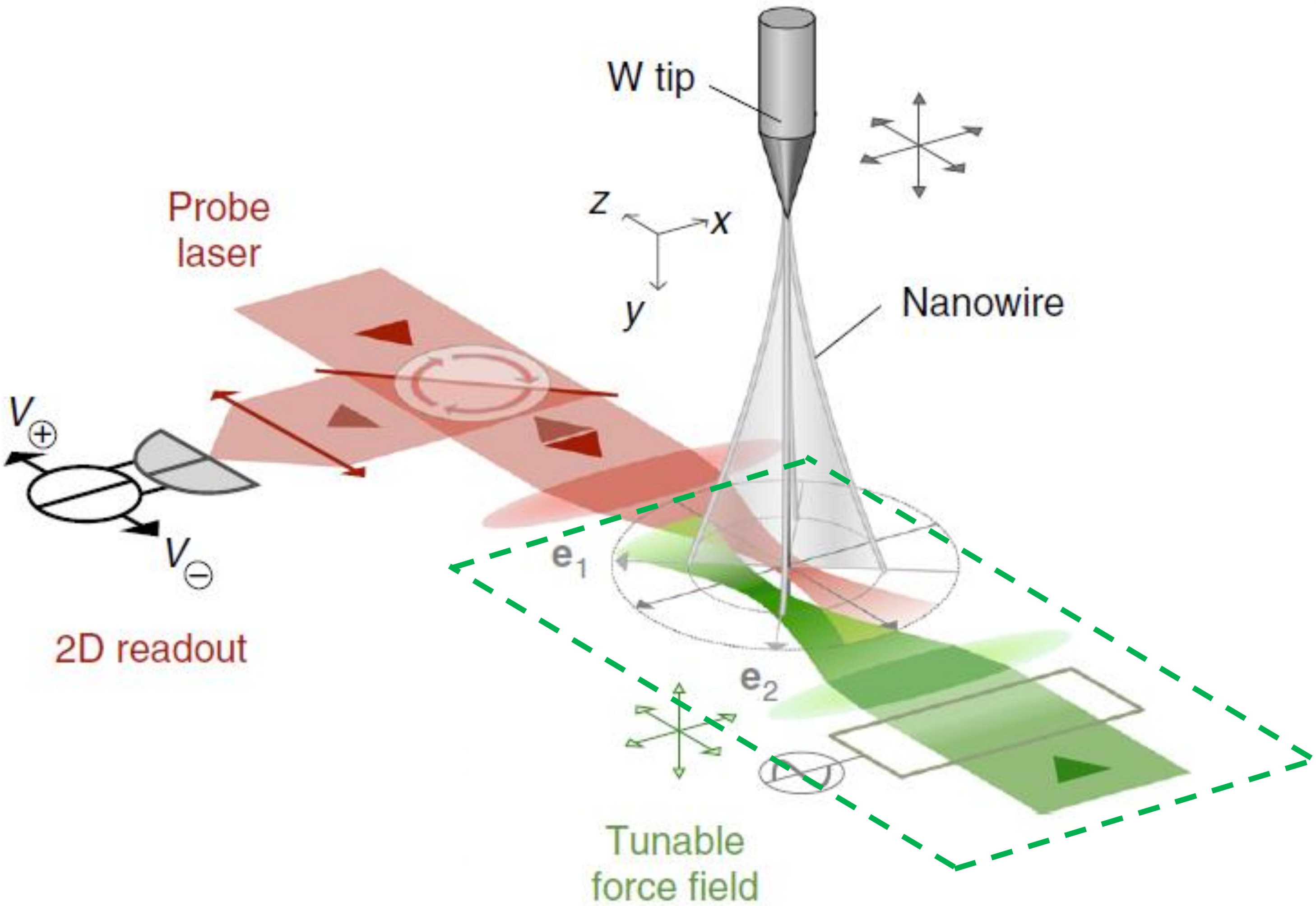
Power spectrum of displacement



Nat. Nanotechnol. **9**, 920(2014).

2-Dim vectorial force sensor

Optical force



External force added

$$\delta\ddot{\boldsymbol{r}} = -\begin{pmatrix}\Omega_1^2 & 0\\ 0 & \Omega_2^2\end{pmatrix}\delta\boldsymbol{r} - \begin{pmatrix}\Gamma_1 & 0\\ 0 & \Gamma_2\end{pmatrix}\delta\dot{\boldsymbol{r}} + \left(\boldsymbol{F}(\boldsymbol{r_0}+\delta\boldsymbol{r}) + \delta\boldsymbol{F_{th}}\right) / M_{eff}$$

$$\boldsymbol{F}(\boldsymbol{r_0}+\delta\boldsymbol{r}) \simeq \boldsymbol{F}(\boldsymbol{r_0}) + (\delta\boldsymbol{r}\cdot\boldsymbol{\nabla})\mathbf{F}|\mathbf{r_0}$$

$$\delta\ddot{\boldsymbol{r}} = -\begin{pmatrix}\Omega_1^2 & 0\\ 0 & \Omega_2^2\end{pmatrix}\delta\boldsymbol{r} - \begin{pmatrix}\Gamma_1 & 0\\ 0 & \Gamma_2\end{pmatrix}\delta\dot{\boldsymbol{r}} + \left(\boldsymbol{F}(\boldsymbol{r_0}) / M_{eff} + (\delta\boldsymbol{r}\cdot\boldsymbol{\nabla})\mathbf{F}|\mathbf{r_0} + \delta\boldsymbol{F_{th}}\right) / M_{eff}$$

$$\delta\ddot{\boldsymbol{r}} = -\begin{pmatrix}\Omega_1^2 & 0\\ 0 & \Omega_2^2\end{pmatrix}\delta\boldsymbol{r} - \begin{pmatrix}\Gamma_1 & 0\\ 0 & \Gamma_2\end{pmatrix}\delta\dot{\boldsymbol{r}} + \begin{pmatrix}g_{11} & g_{21}\\ g_{12} & g_{22}\end{pmatrix}\delta\boldsymbol{r} + \delta\boldsymbol{F_{th}}/\boldsymbol{M_{eff}}$$

$$g_{ij} \equiv \frac{1}{M_{eff}}\partial_i F_j(r_0)$$

$$\delta\ddot{\boldsymbol{r}} = -\begin{pmatrix}\Omega_1^2 - g_{11} & -g_{21}\\ g_{12} & \Omega_2^2 - g_{22}\end{pmatrix}\delta\boldsymbol{r} - \begin{pmatrix}\Gamma_1 & 0\\ 0 & \Gamma_2\end{pmatrix}\delta\dot{\boldsymbol{r}} + \delta\boldsymbol{F_{th}}/\boldsymbol{M_{eff}}$$

External force added

$$\delta\boldsymbol{r}[\Omega] \equiv \chi[\Omega]\delta\boldsymbol{F_{th}}[\Omega]$$

$$\delta\boldsymbol{r}[\Omega] \equiv \int \delta\boldsymbol{r}(t)e^{-i\Omega t}dt$$

$$\chi^{-1}[\Omega] = M_{eff}\begin{pmatrix} \Omega_1^2 - \Omega^2 - i\Gamma\Omega - g_{11} & -g_{21} \\ -g_{12} & \Omega_2^2 - \Omega^2 - i\Gamma\Omega - g_{22} \end{pmatrix}$$

Langevian noise spectrum

$$S_{\delta r_\beta}[\Omega] = \frac{S_{F_{th}}}{|\Xi[\Omega]|^2}\left\{\begin{matrix} \cos^2\beta\left((\Omega_2^2 - \Omega^2 - g_{22})^2 + \Omega^2\Gamma^2 + g_{21}^2\right) \\ +\sin^2\beta\,(g_{12}^2 + (\Omega_1^2 - \Omega^2 - g_{11})^2 + \Omega^2\Gamma^2) \\ +\sin 2\beta\,(g_{12}(\Omega_2^2 - \Omega^2 - g_{22}) + g_{21}(\Omega_1^2 - \Omega^2 - g_{11})) \end{matrix}\right\}$$

$\Xi[\Omega] = M_{eff}\{(\Omega_1^2 - \Omega^2 - i\Gamma\Omega - g_{11})(\Omega_2^2 - \Omega^2 - i\Gamma\Omega - g_{22}) - g_{12}g_{21}\}$

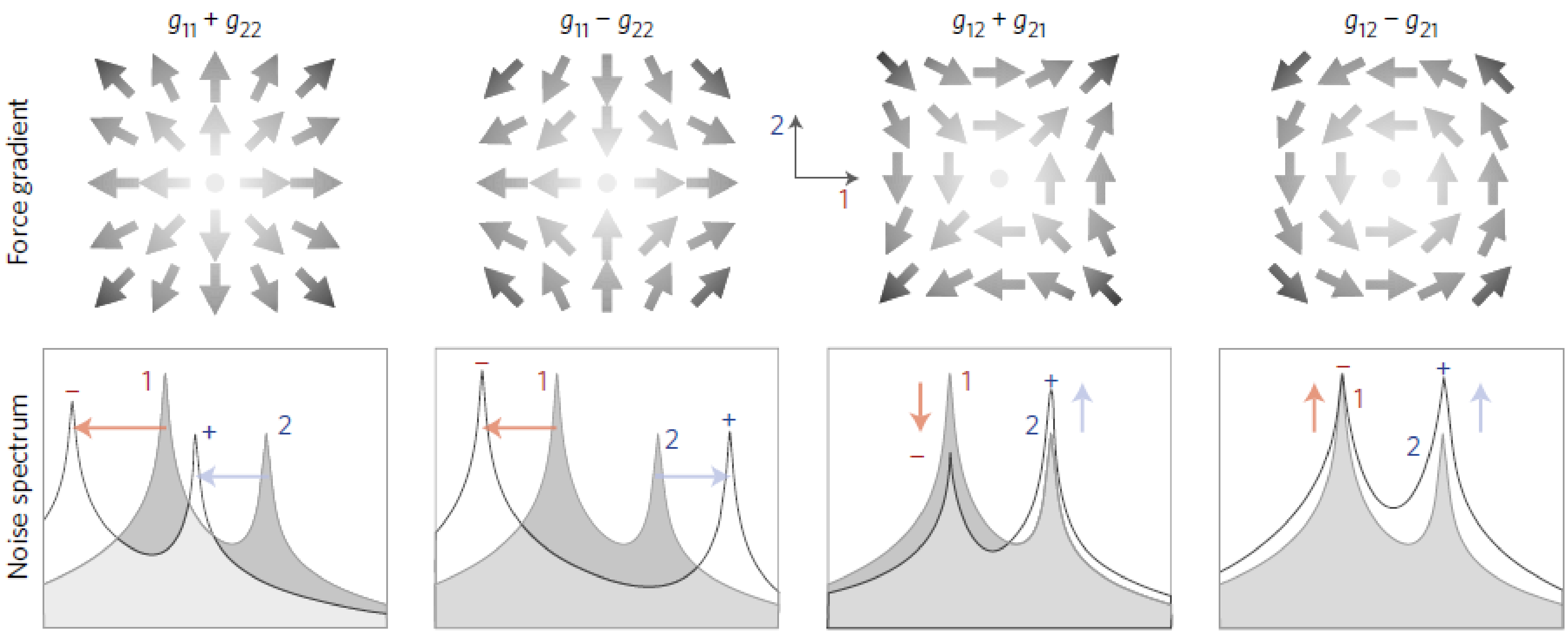
$\Gamma = \Gamma_1 \simeq \Gamma_2$

$\boldsymbol{e_\beta}$: measurement vector

$\beta \equiv \cos^{-1}(\boldsymbol{e_1}\cdot\boldsymbol{e_\beta})$

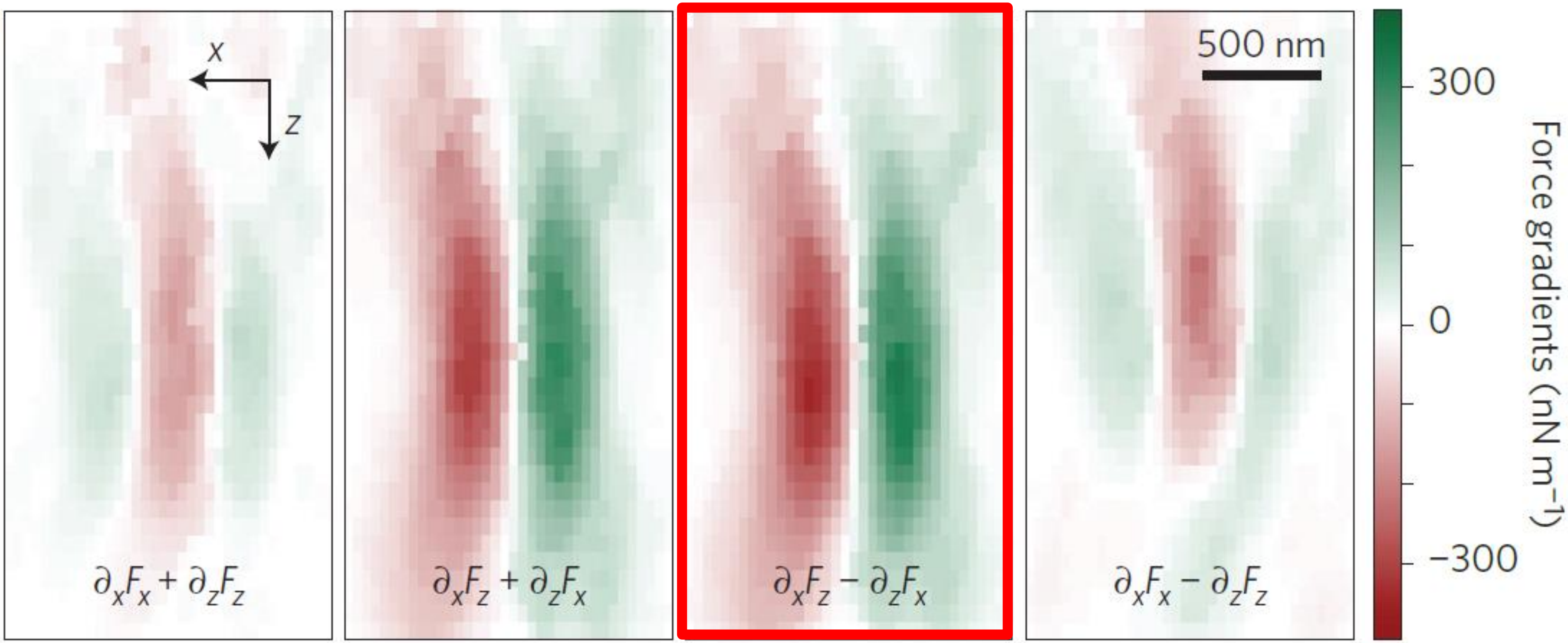
Force dependent spectrum

$\partial_x F_x + \partial_z F_z$ $\partial_x F_x - \partial_z F_z$ $\partial_x F_z + \partial_z F_x$ $\partial_x F_z - \partial_z F_x$



Nat. Nanotechnol. **12**, 156 (2016).

Non-conservative optical force



$$\nabla \times \boldsymbol{F} \neq 0$$

Nat. Nanotechnol. **9**, 920(2014).

Exceptional point of two normal modes

Interaction via non-conservative force

$$\delta\ddot{\boldsymbol{r}} = -\begin{pmatrix}\Omega_1^2 & 0\\ 0 & \Omega_2^2\end{pmatrix}\delta\boldsymbol{r} - \begin{pmatrix}\Gamma_1 & 0\\ 0 & \Gamma_2\end{pmatrix}\delta\dot{\boldsymbol{r}} + \left(\boldsymbol{F}(\boldsymbol{r_0} + \delta\boldsymbol{r})\right) / M_{eff}$$

$$\boldsymbol{F}(\boldsymbol{r_0} + \delta\boldsymbol{r}) \simeq \boldsymbol{F}(\boldsymbol{r_0}) + (\delta\boldsymbol{r}\cdot\boldsymbol{\nabla})\mathbf{F}|_{\mathbf{r_0}}$$

$$\delta\ddot{\boldsymbol{r}} = -\begin{pmatrix}\Omega_1^2 & 0\\ 0 & \Omega_2^2\end{pmatrix}\delta\boldsymbol{r} - \begin{pmatrix}\Gamma_1 & 0\\ 0 & \Gamma_2\end{pmatrix}\delta\dot{\boldsymbol{r}} + \left(\boldsymbol{F}(\boldsymbol{r_0}) / M_{eff} + (\delta\boldsymbol{r}\cdot\boldsymbol{\nabla})\mathbf{F}|_{\mathbf{r_0}}\right) / M_{eff}$$

$$g_{ij} \equiv \frac{1}{M_{eff}}\partial_i F_j(r_0)$$

$$\delta\ddot{\boldsymbol{r}} = -\begin{pmatrix}\Omega_1^2 - g_{11} & -g_{21}\\ -g_{12} & \Omega_2^2 - g_{22}\end{pmatrix}\delta\boldsymbol{r} - \begin{pmatrix}\Gamma_1 & 0\\ 0 & \Gamma_2\end{pmatrix}\delta\dot{\boldsymbol{r}}$$

$$\begin{pmatrix}\Omega_1^2 - \Omega^2 - i\Gamma_1\Omega - g_{11} & -g_{21}\\ -g_{12} & \Omega_2^2 - \Omega^2 - i\Gamma_2\Omega - g_{22}\end{pmatrix}\delta\boldsymbol{r}[\Omega] = 0$$

Interaction via non-conservative force

Dressed eigenfrequency

$$\Omega_{\pm}^2 \equiv \frac{\Omega_2^2 + \Omega_1^2 + g_{22} + g_{11}}{2} - i\Gamma \pm \frac{1}{2}\sqrt{((\Omega_2^2 - \Omega_1^2 - g_{22} + g_{11})^2 + 4g_{12}g_{21}}$$

Dressed eigenmodes

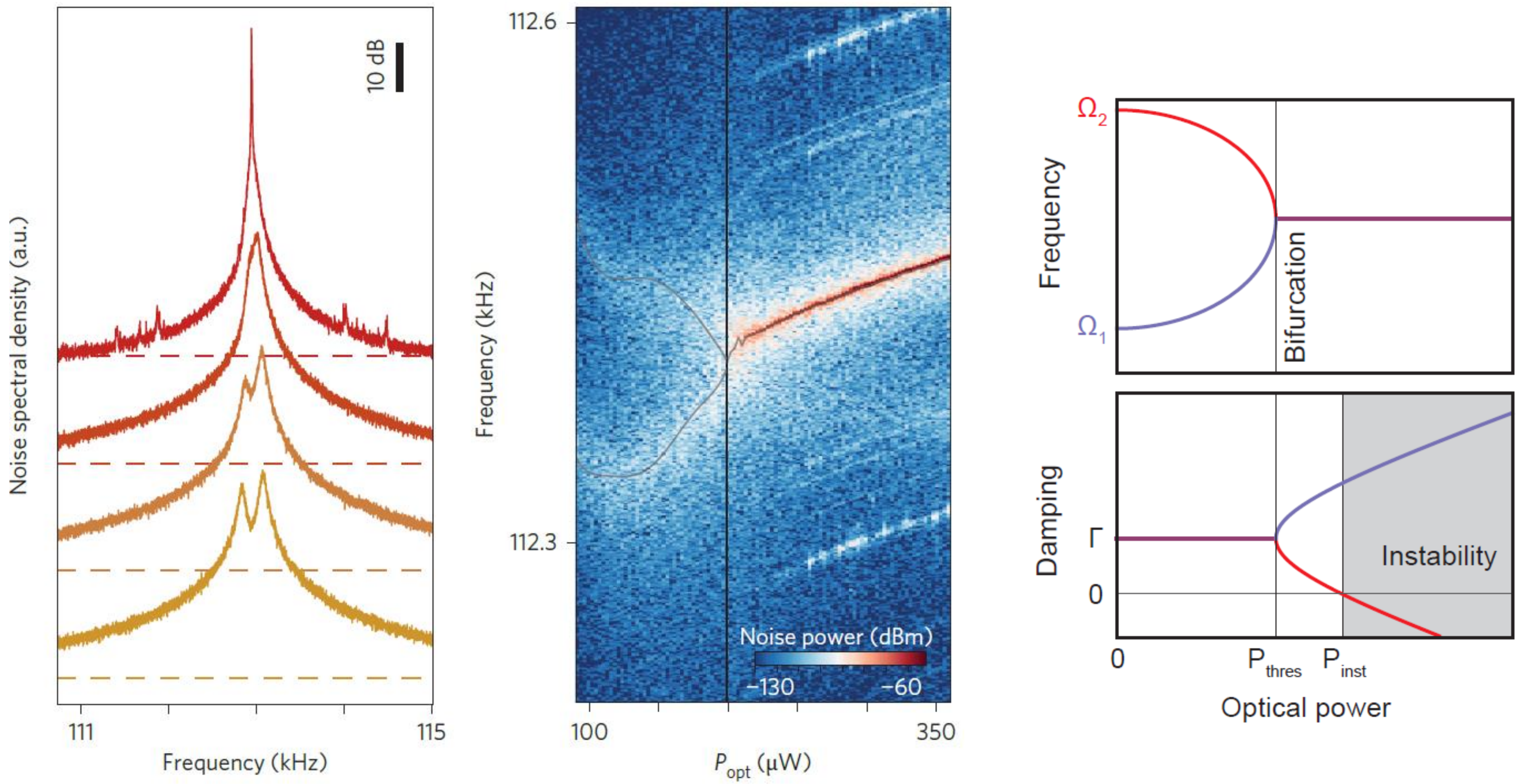
$$\boldsymbol{e}_{-} \equiv \frac{1}{\sqrt{g_{12}^2 + \Omega_2^2 - g_{22} - \Omega_{-}^2}} \begin{pmatrix} \Omega_2^2 - g_{22} - \Omega_{-}^2 \\ g_{12} \end{pmatrix}$$

$$g_{ij} \equiv \frac{1}{M_{eff}} \partial_i F_j(r_0)$$

$$\boldsymbol{e}_{+} \equiv \frac{1}{\sqrt{g_{21}^2 + \Omega_2^2 - g_{22} - \Omega_{-}^2}} \begin{pmatrix} -g_{21} \\ \Omega_2^2 - g_{22} - \Omega_{-}^2 \end{pmatrix}$$

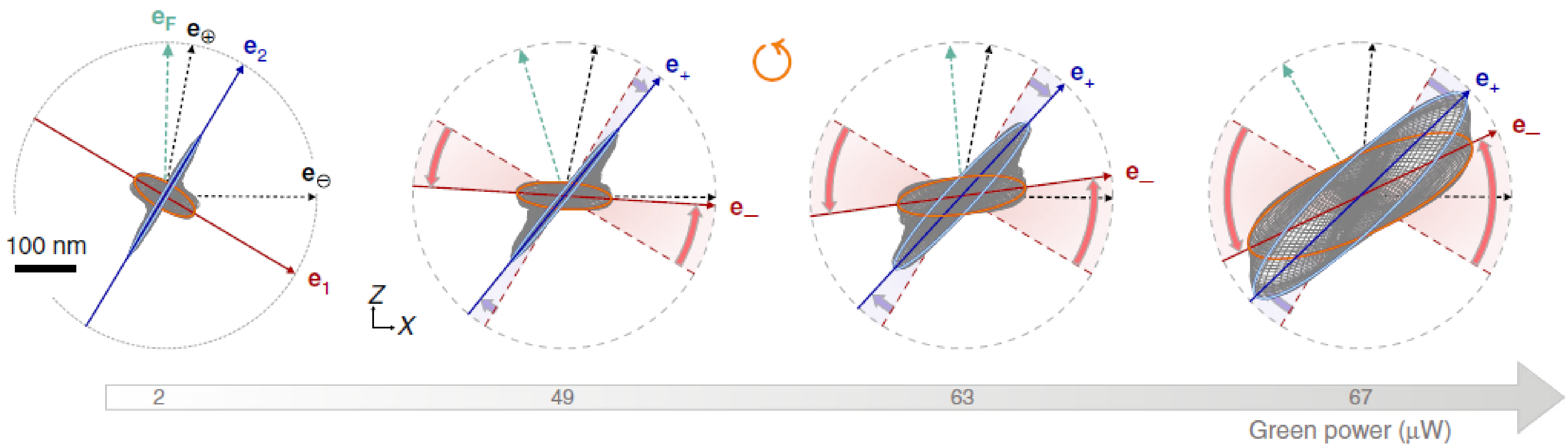
$\boldsymbol{e}_{-} \cancel{\perp} \boldsymbol{e}_{+}$ if $\boldsymbol{\nabla} \times \boldsymbol{F} \neq 0(\, g_{12} \neq g_{21})$

Exceptional point



Nat. Nanotechnol. **9**, 920(2014).

Eigenmode direction

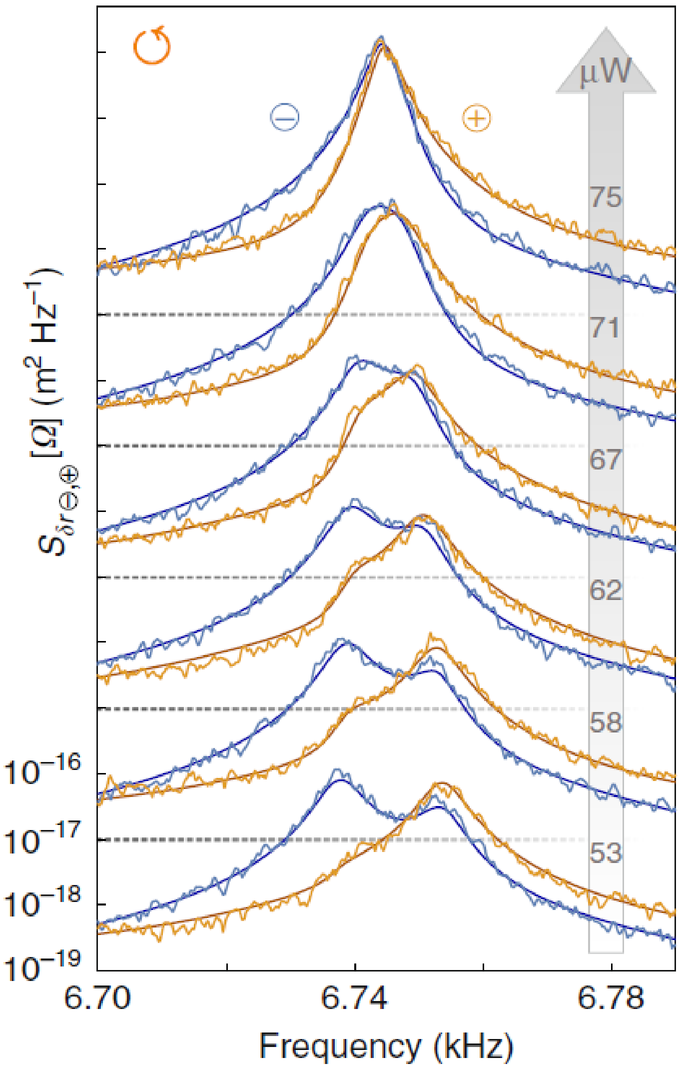


Eigenmodes are not orthogonal.

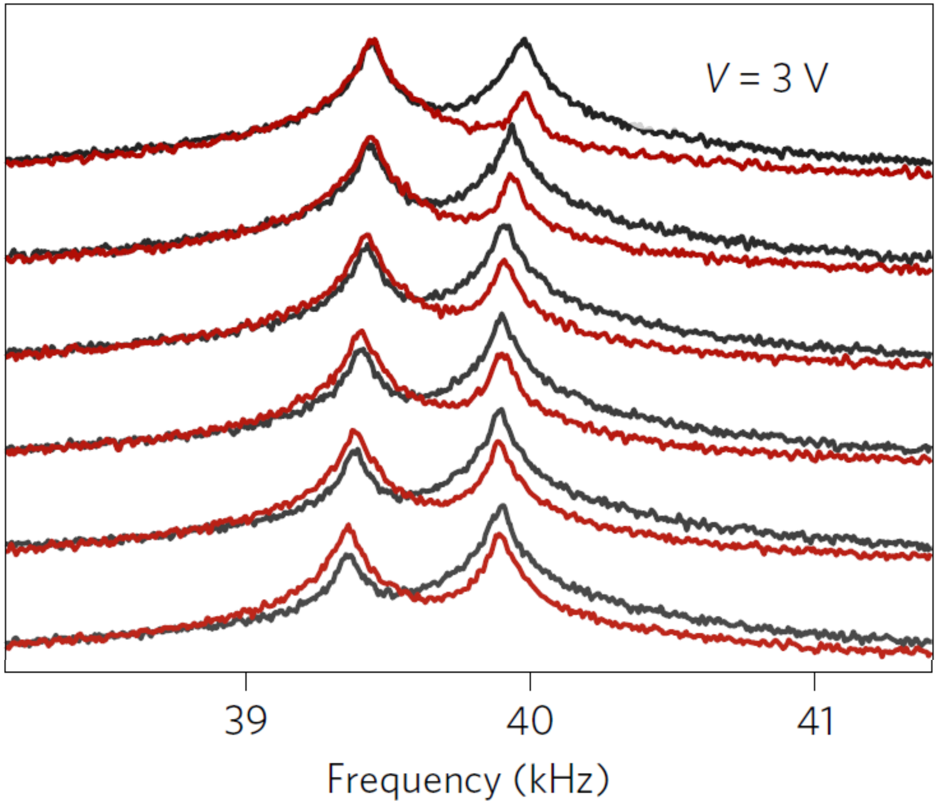
Eigenmodes converge towards each other.

Polarizations are converted (linear to circular).

Distortion of spectrum



Non-conservative force



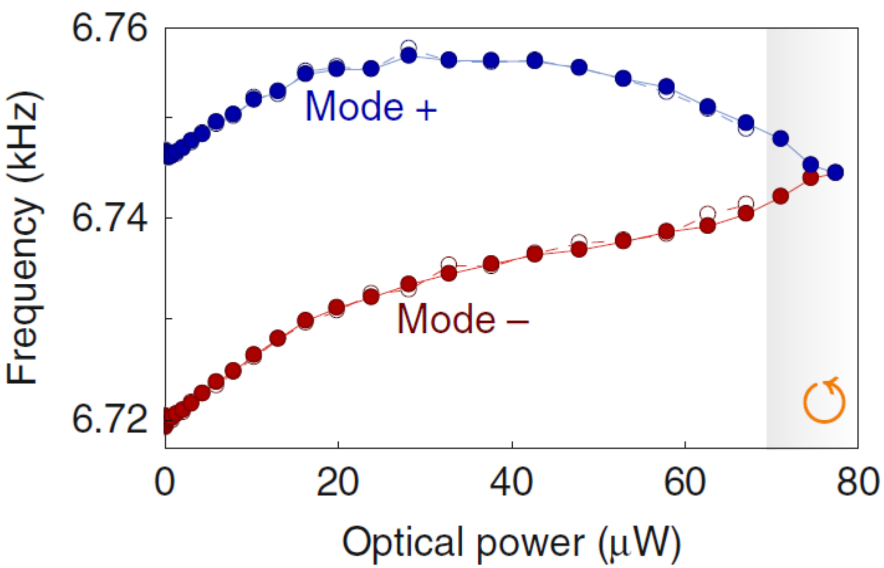
Conservative force

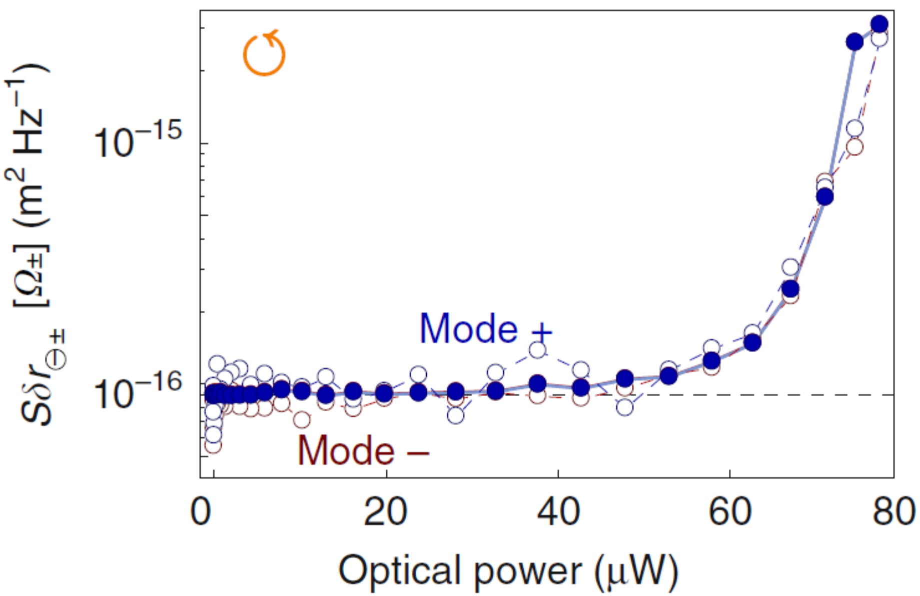
Nat. Nanotechnol. **12**, 156 (2016).

Not well fitted with Lorentzian shape.

Peaks are asymmetric.

Excess noise at EP



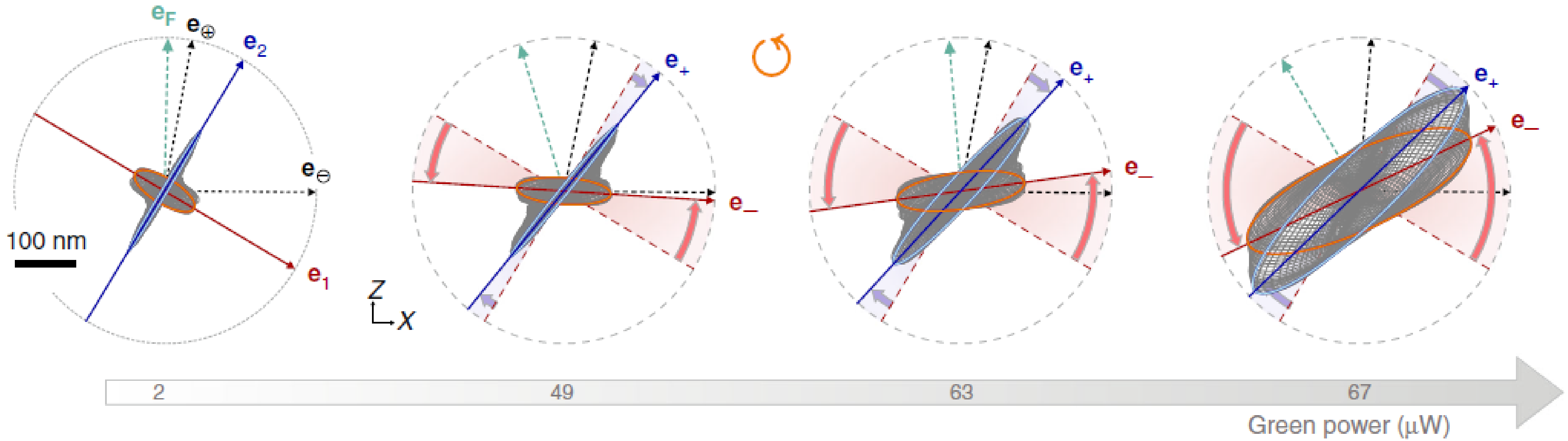


$$\boxed{S_{\delta r_{\theta_\pm}}[\Omega] = \frac{2k_BT}{\Omega} Im\chi_{\pm\pm}[\Omega]} \left(1 + \frac{(g_{21}-g_{12})^2}{\left(\Omega_\mp^2 - \Omega^2\right)^2 + \Omega^2\Gamma^2}\right)$$

Fluctuation-Dissipation Theorem Enhancement factor

$$\frac{2k_BT}{\Omega} Im\chi_{\pm\pm}[\Omega] = \frac{2\Gamma k_BT}{M_{eff}\left(\left(\Omega_\pm^2 - \Omega^2\right)^2 + \Omega^2\Gamma^2\right)}$$

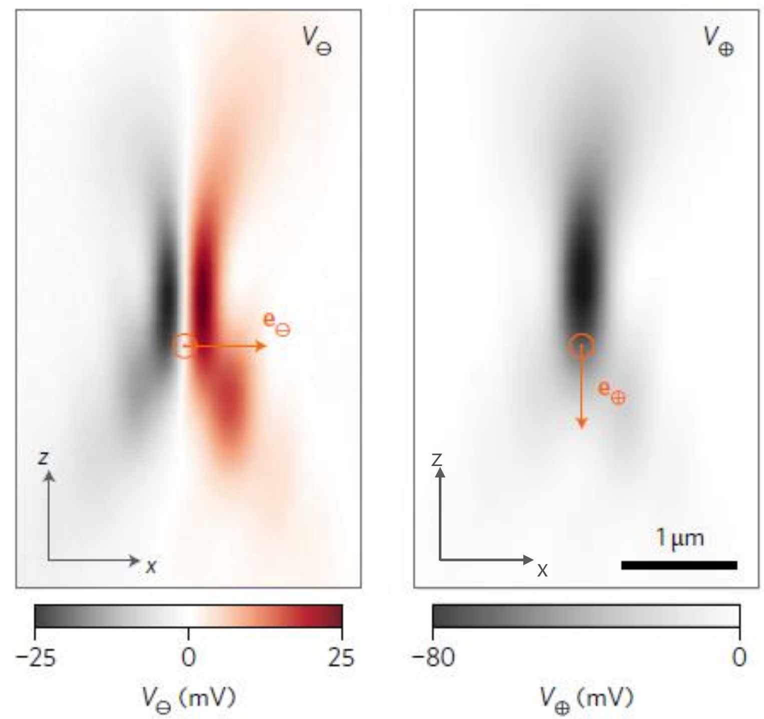
Response measurements



Q & A

Q : Compare the resolutions of the displacement measurement along x and z direction.

A : From the overall structure of voltage signals, it seem that the resolution along x direction should be better than the one along z direction. However, the resolution depends on the gradient and the dip in V+ signal is deeper; it is hard to decide it with given data.



Q : Is there any possibility that enhancement of the response is a consequence of the increased optical power?

A : Theoretically, the constant force only shift the equilibrium position of the nanowire. However, nonlinearity threshold is very close to the bifurcation point. They should measure the response in the region between the bifurcation point and nonlinearity threshold. If it decrease after the EP, then it can be confirmed.