Selective enhancement of topologically induced interface states in a dielectric resonator chain

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Group information

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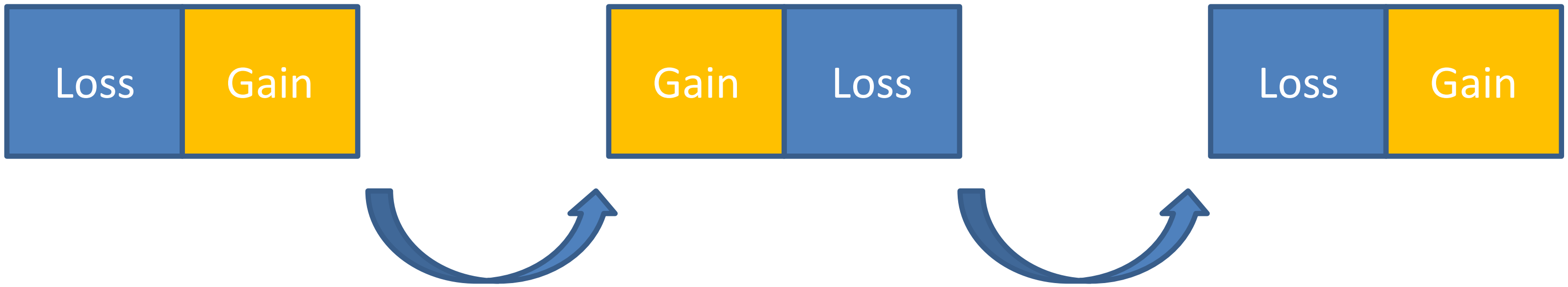
- 2 Ph.D students in his group

Research interests

- Quantum transport including graphene
  – e.g. topological insulators
- Quantum optics, microlasers and photonics
  – PT-symmetric system, topologically protected states
- Quantum dynamical systems including atomic systems
  – Characterizing resonances in complex open systems

PT symmetric system



Parity ($\mathbf{x} \to -\mathbf{x}$) Time reversal ($i \to -i$)

PT symmetric Hamiltonian: $(\mathrm{PT})H(\mathrm{PT}) = H$

$$H\phi_n = E_n\phi_n$$

$$H(\mathrm{PT})\phi_n = (\mathrm{PT})E_n\phi_n = \begin{cases} E_n(\mathrm{PT})\phi_n & \text{: PT-exact phase (real eigenvalue)} \\ E_n^*(\mathrm{PT})\phi_n & \text{: PT-broken phase (complex eigenvalue)} \end{cases}$$

(Example)

$$H = \begin{pmatrix} iV & g \\ g & -iV \end{pmatrix} \Longrightarrow E_\pm = \pm\sqrt{g^2 - V^2}$$

EP condition: $g = V$

- At an EP, transition between two phases occurs.

Topologically protected states

- Tight-binding Hamiltonian with disorder

$$\hat{H} = \begin{pmatrix} \ddots & \tilde{c}_1^2 & 0 & 0 & 0 & 0 & 0 \\ \tilde{c}_1^2 & -i\gamma & \tilde{c}_2^1 & 0 & 0 & 0 & 0 \\ 0 & \tilde{c}_2^1 & i\gamma & \tilde{c}_1^1 & 0 & 0 & 0 \\ 0 & 0 & \tilde{c}_1^1 & 0 & \tilde{c}_1^1 & 0 & 0 \\ 0 & 0 & 0 & \tilde{c}_1^1 & -i\gamma & \tilde{c}_2^1 & 0 \\ 0 & 0 & 0 & 0 & \tilde{c}_2^1 & i\gamma & \tilde{c}_1^2 \\ 0 & 0 & 0 & 0 & 0 & \tilde{c}_1^2 & \ddots \end{pmatrix}$$

$\tilde{c}_1, \tilde{c}_2$: modulated coupling by disorder
γ: decay rate

- Define the operators:

$$\hat{\sigma}_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & \ddots \end{pmatrix}, \; \hat{\sigma}_x = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ \iddots & 0 & 0 \end{pmatrix} \Rightarrow \hat{\sigma}_x \hat{H}^\dagger \hat{\sigma}_x = \hat{H}$$

($[\hat{H}, \mathcal{PT}] = 0$, PT-symmetric)

- Then,

$$\det\left(\hat{H}\right) = \det\left(\hat{\sigma}_x \hat{H}^\dagger \hat{\sigma}_x\right) = \det\left(\hat{H}^\dagger\right)$$

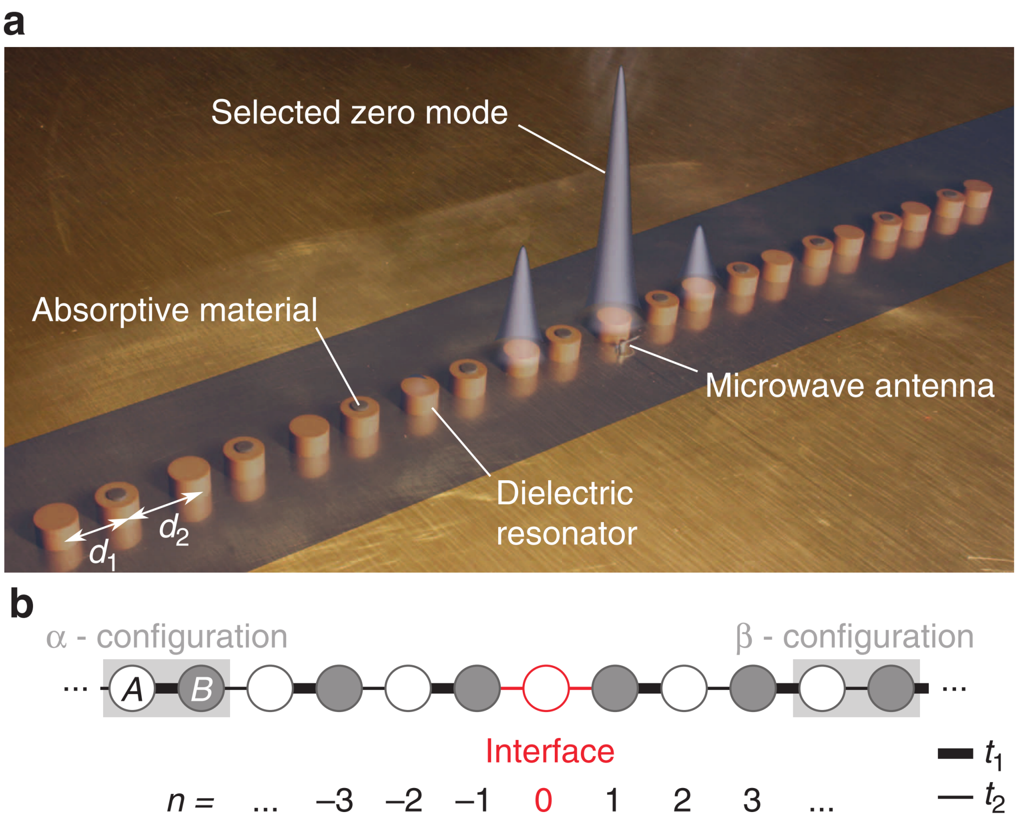
$$\det\left(\hat{H}\right) = (-1)^{2\cdot(N-1)/2} \det\left(-\hat{H}^\dagger\right) = \det\left(-\hat{H}^\dagger\right) = (-1)^N \det\left(\hat{H}^\dagger\right)$$

($\hat{\sigma}_x \hat{\sigma}_z \hat{H} \hat{\sigma}_z \hat{\sigma}_x = -\hat{H}$)

$$\Rightarrow \boxed{\det\left(\hat{H}\right) = (-1)^N \det\left(\hat{H}\right)} \Rightarrow \det\left(\hat{H}\right) = 0.$$

(N=odd in this system, **zero-eigenvalue always exists**)

Realization of dimer chains



- **(a)**: Picture of experimental microwave resonator chain
- **(b)**: Schematic of the chain. A (white) and B (grey) indicate the sublattice. t_1 and t_2 denote the coupling strength.
- Coupling controlled by varying the spacing between resonators.

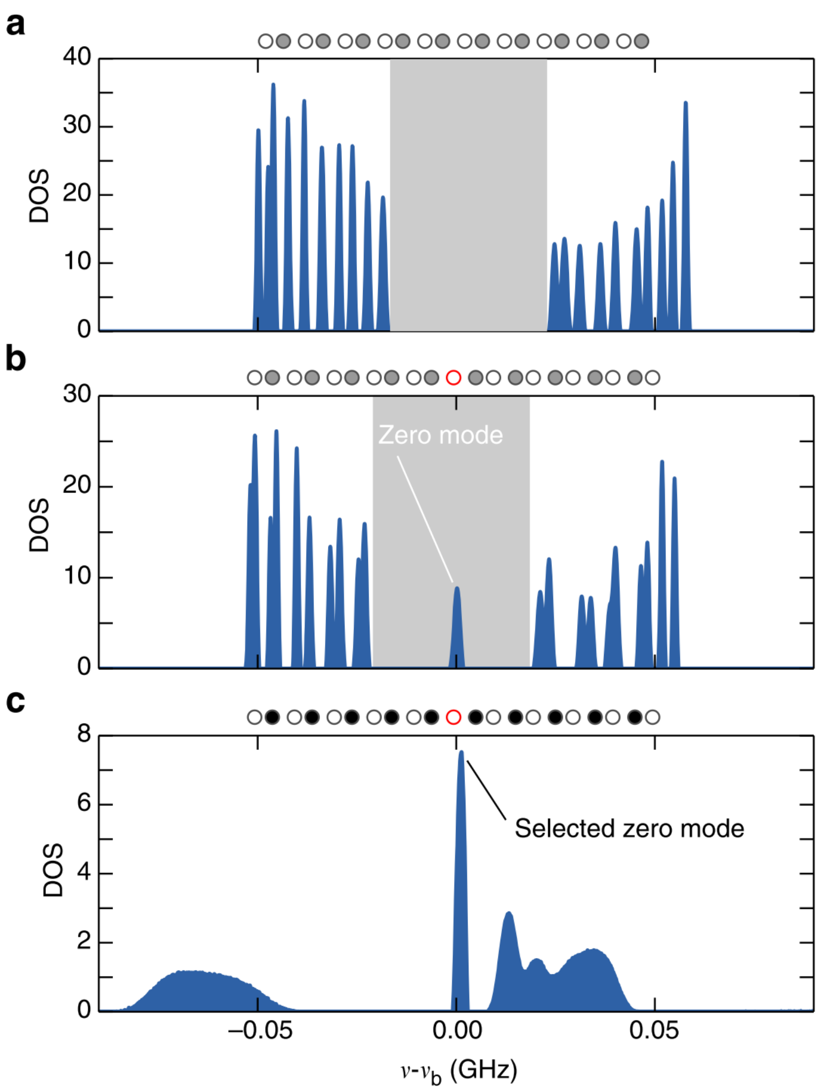
Described by tight-binding equations:

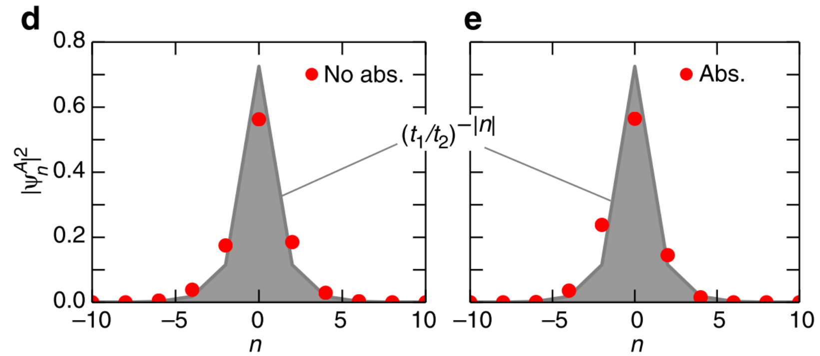
$$(\alpha)\begin{cases}(\nu-\nu_n)\psi_n = t_2\psi_{n-1}+t_1\psi_{n+1}, & n=-2,-4,\ldots\\ (\nu-\nu_n)\psi_n = t_1\psi_{n-1}+t_2\psi_{n+1}, & n=-1,-3,\ldots,\end{cases}$$

$$\text{interface}\,\{(\nu-\nu_0)\psi_0=t_2(\psi_{-1}+\psi_1), \qquad n=0,$$

$$(\beta)\begin{cases}(\nu-\nu_n)\psi_n = t_2\psi_{n-1}+t_1\psi_{n+1}, & n=1,3,\ldots,\\ (\nu-\nu_n)\psi_n = t_1\psi_{n-1}+t_2\psi_{n+1}, & n=2,4,\ldots,\end{cases}$$

Density of states and zero-mode profiles

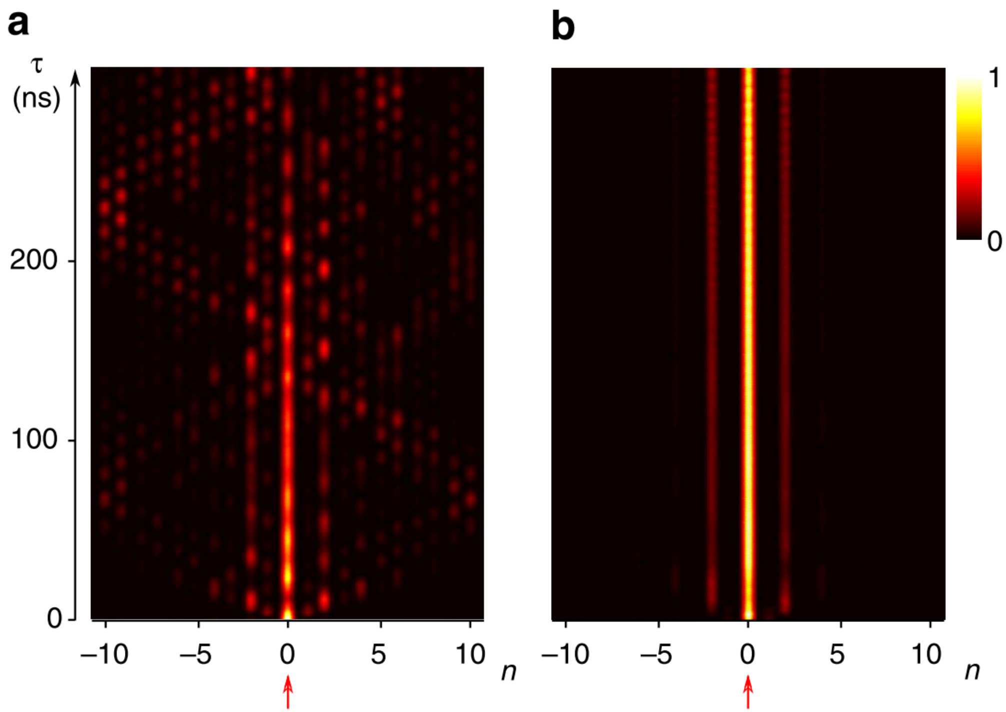




- **(a)** DOS for a defectless chain
- **(b)** DOS with a defect
- **(c)** DOS with a defect and absorption on B sites
- **(d)** Intensity profile of zero-mode on A sites
- **(e)** Intensity profile of zero-mode on A sites with absorption on B sites

➔ **Visibility of the zero mode enhanced in frequency domain**

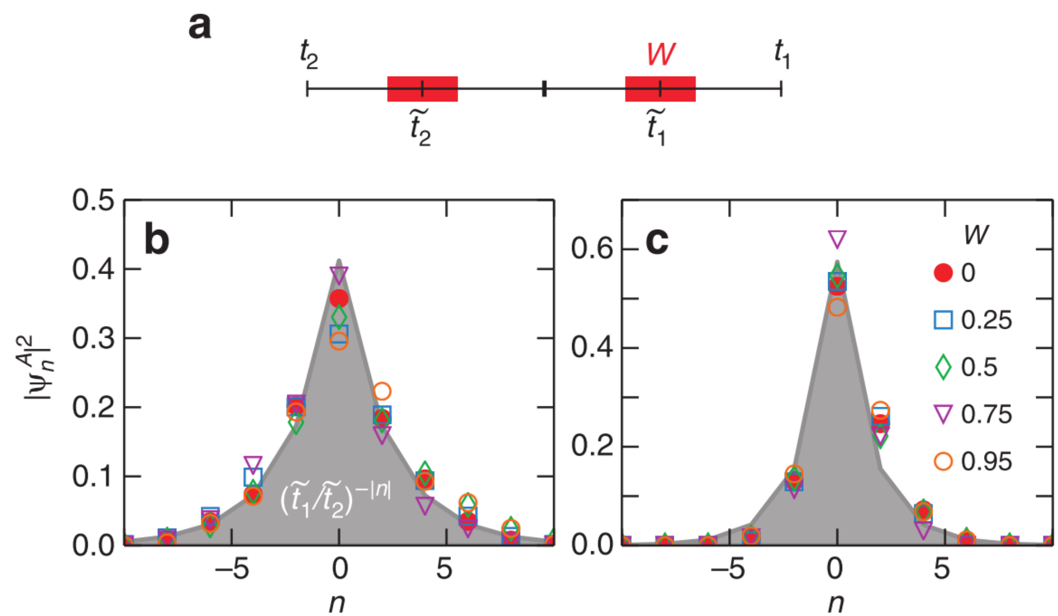
Selective enhancement of the zero mode



- **(a)** Time evolution of the field intensity when pulse launched on the defect site without absorption on B sites.
- **(b)** With absorption on B sites. It shows that adding losses drastically enhances the visibility of the zero-mode.

➔ Visibility enhanced in time domain

Robustness of the zero mode against structural disorder



- **(a)** Randomized coupling strengths ($\tilde{t}_1, \tilde{t}_2$) and disorder strength (W)
- **(b)** Zero-mode intensity profiles on A sites without losses
- **(c)** Zero-mode intensity profiles with losses on the B sites.

➔ Zero mode is invariant under random variation of coupling strength, which shows the robustness against structural disorder

Summary

- Topologically induced defect state in a chain of dielectric microwave cavity resonators realized.
- It was shown that visibility of the defect state could be enhanced both in frequency and time domain by imposing losses.
- This mode selection mechanism carries over wide range of topological and PT-symmetric optical platforms.