Emergence, Coalescence, and Topological Properties of Multiple Exceptional Points and Their Experimental Realization

2016-05-10 Journal club

Kim, Soyun

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PHYSICAL REVIEW X 6, 021007 (2016)

Emergence, Coalescence, and Topological Properties of Multiple Exceptional Points and Their Experimental Realization

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(Received 31 December 2015; revised manuscript received 5 March 2016; published 12 April 2016)

Non-Hermitian systems distinguish themselves from Hermitian systems by exhibiting a phase transition point called an exceptional point (EP), at which two eigenstates coalesce under a system parameter variation. Many interesting EP phenomena, such as level crossings in nuclear and condensed matter physics, and unusual phenomena in optics, such as loss-induced lasing and unidirectional transmission, can be understood by considering a simple 2×2 non-Hermitian matrix. At a higher dimension, more complex EP physics not found in two-state systems arises. We consider the emergence and interaction of multiple EPs in a four-state system theoretically and realize the system experimentally using four coupled acoustic cavities with asymmetric losses. We find that multiple EPs can emerge, and as the system parameters vary, these EPs can collide and merge, leading to higher-order singularities and topological characteristics much richer than those seen in two-state systems. The new physics obtained is not limited to the acoustic systems demonstrated here. It also applies to other systems as well, such as coupled photonic cavities and waveguides.

DOI: 10.1103/PhysRevX.6.021007 Subject Areas: Acoustics, Materials Science

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Contents

- Two coupled acoustic cavity resonators
- Four-state non-Hermitian Hamiltonian with coupling
- Eigenfrequency phase diagram and experiment realizations
- Topological characteristics around singularities
- Conclusions

Two coupled acoustic cavity resonators

- Two-state system of coupled cavities

$$H = \begin{pmatrix} \omega_2 - i\Gamma_0 & \kappa \\ \kappa & \omega_2 - i\Gamma \end{pmatrix}$$

$$\Gamma = \Gamma_0 + \boxed{\Delta\Gamma}$$

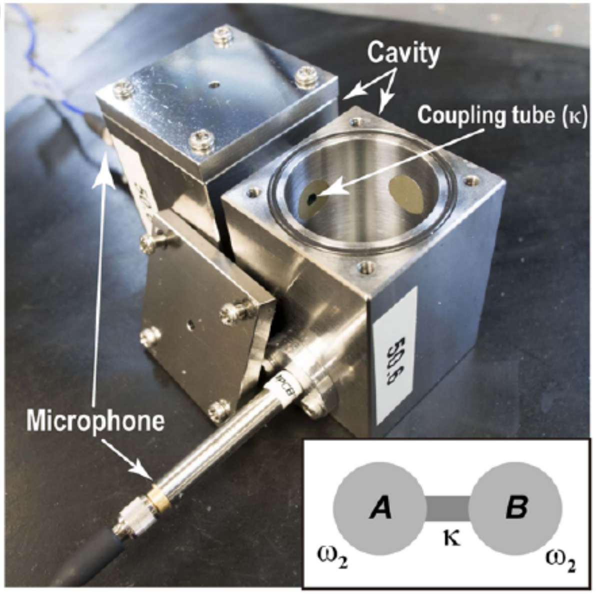
Intrinsic loss tunable loss

eigenfrequency $$\tilde{\omega}_{1,2} = \omega_2 - i\frac{\Gamma + \Gamma_0}{2} \pm \frac{1}{2}\sqrt{4\kappa^2 - (\Delta\Gamma)^2}$$

- Acoustic resonant cavities

κ	coupling tubes (cross-sectional area)
ω_2	Varying depth ($\omega_2 = \frac{\pi \upsilon}{h}$, υ : speed of sound in air)
$\Delta\Gamma$	Inserting the assembly of sponge and putty

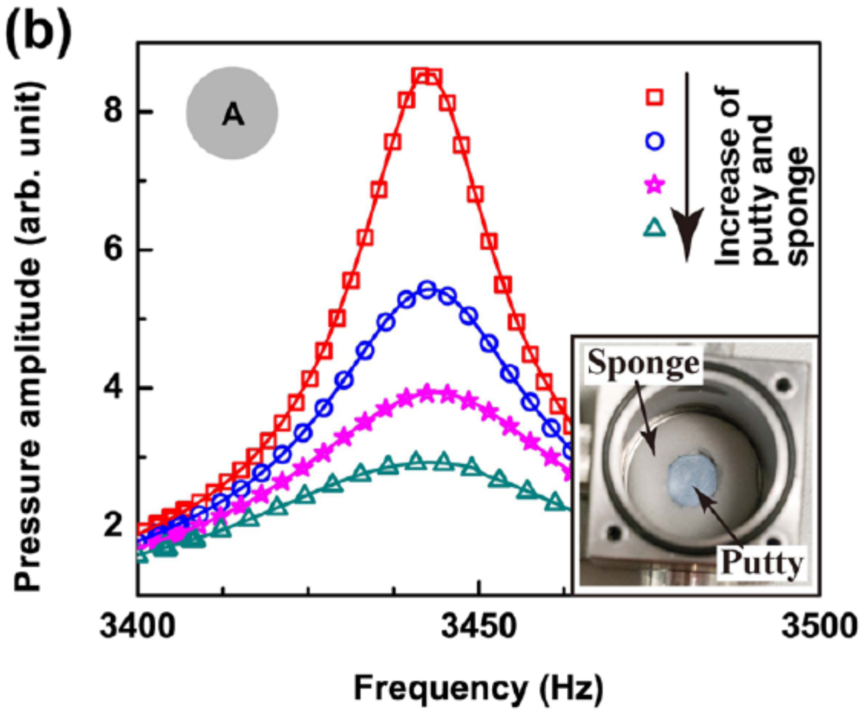
(a)



Cylindrical metallic cavities (Stainless steel)
Radius : 15.0mm
Air : 1atm
Temperature : 295K

Two coupled acoustic cavity resonators

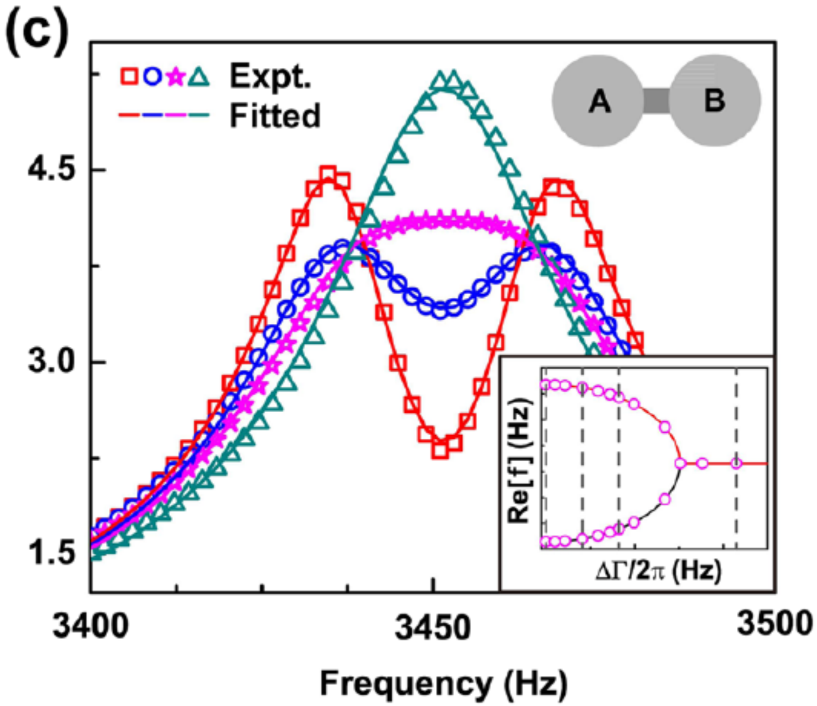
Measured pressure responses of a single cavity



- $h = 50.6$ mm
- Red open squares is the case of no loss in the cavity.
- Fitting data with $|P(\omega)| \propto \left|\frac{1}{\omega-(\omega_2-i\Gamma)}\right|$ to find resonant frequency ω_2, intrinsic loss Γ_0, additional loss $\Delta\Gamma$

Two coupled acoustic cavity resonators

Measured pressure response spectra of the two coupled cavities



- Pumping and measurements are performed at cavity A and asymmetric loss is provided in cavity B.
- $\kappa \rightarrow 2.0\text{mm}, 0.8\text{mm}$ tubes
- Green function of the system

$$\overleftrightarrow{G}(\omega) = \sum_{j=1}^{N} \frac{|\tilde{\phi}_j^R\rangle\langle\tilde{\phi}_j^L|}{\omega - \tilde{\omega}_j}$$

$$|P(\omega)| = A\left|\langle p|\overleftrightarrow{G}(\omega)|s\rangle\right|$$

$|s\rangle$: source

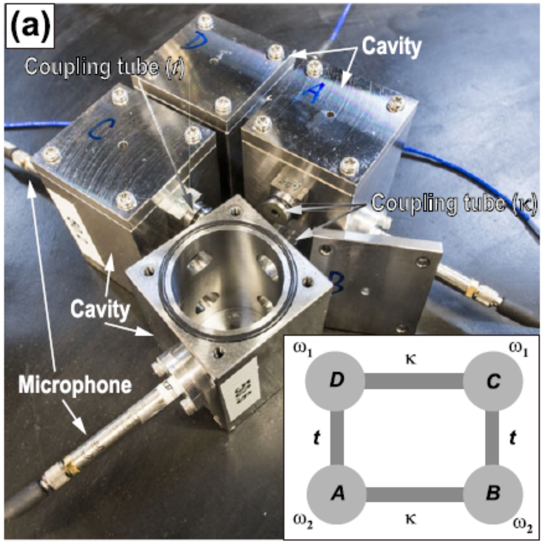
$|p\rangle$: probe

- Fitting steps

1. To obtain ω_2, Γ_0, and κ
2. To obtain $\Delta\Gamma$ using ω_2, Γ_0, and κ

- Two eigenmodes coalesce at an EP when $\Delta\Gamma = 2|\kappa|$

Four-state non-Hermitian Hamiltonian with coupling



$$H = \begin{pmatrix} \omega_2 - i\Gamma_0 & \kappa & 0 & t \\ \kappa & \omega_2 - i\Gamma & t & 0 \\ 0 & t & \omega_1 - i\Gamma_0 & \kappa \\ t & 0 & \kappa & \omega_1 - i\Gamma \end{pmatrix}$$

eigenfrequency $\tilde{\omega}_j = \omega_0 - i\frac{\Gamma + \Gamma_0}{2} \pm \frac{1}{2}\sqrt{\Delta_1 \pm 4\sqrt{\Delta_2}}$

$$\Delta_1 = -(\Delta\Gamma)^2 + 4\kappa^2 + 4t^2 + (\Delta\omega)^2$$

$$\Delta_2 = 4\kappa^2 t^2 + \kappa^2(\Delta\omega)^2 - (\Delta\Gamma)^2\frac{(\Delta\omega)^2}{4}$$

$$\omega_0 = \frac{\omega_1 + \omega_2}{2},\ \Delta\omega = \omega_1 - \omega_2$$

$j = 1$	$(-,+)$
$j = 2$	$(-,-)$
$j = 3$	$(+,-)$
$j = 4$	$(+,+)$

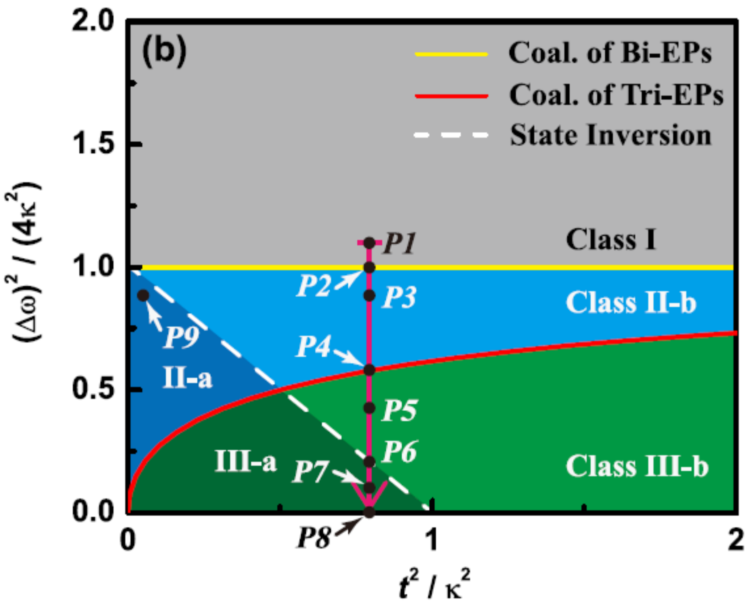
- Conditions for coalescence of states (CS)

$\mathrm{CS} - 1_{\pm} : \Delta_1 \pm 4\sqrt{\Delta_2} = 0,\ \Delta_1 \neq 0,\ \Delta_2 \neq 0$ EP with one state defective

$\mathrm{CS} - 2 : \Delta_2 = 0,\ \Delta_1 \neq 0$ Two different EPs occur simultaneously and each has one state defective

$\mathrm{CS} - 3 : \Delta_1 = \Delta_2 = 0$ Coalescence of four states with three states defective

Eigenfrequency phase diagram



- Each class represents a distinct exceptional point formation pattern (EPFP)
- Solid yellow line marks the coalescence of two EPs

$$\mathrm{CS}-1_{-}: \Delta_1 - 4\sqrt{\Delta_2} = 0, \Delta_1 \neq 0, \Delta_2 \neq 0$$

$$t = 0 \text{ or } (\Delta\omega)^2 = 4\kappa^2$$

- Solid red curve marks the coalescence of three EPs

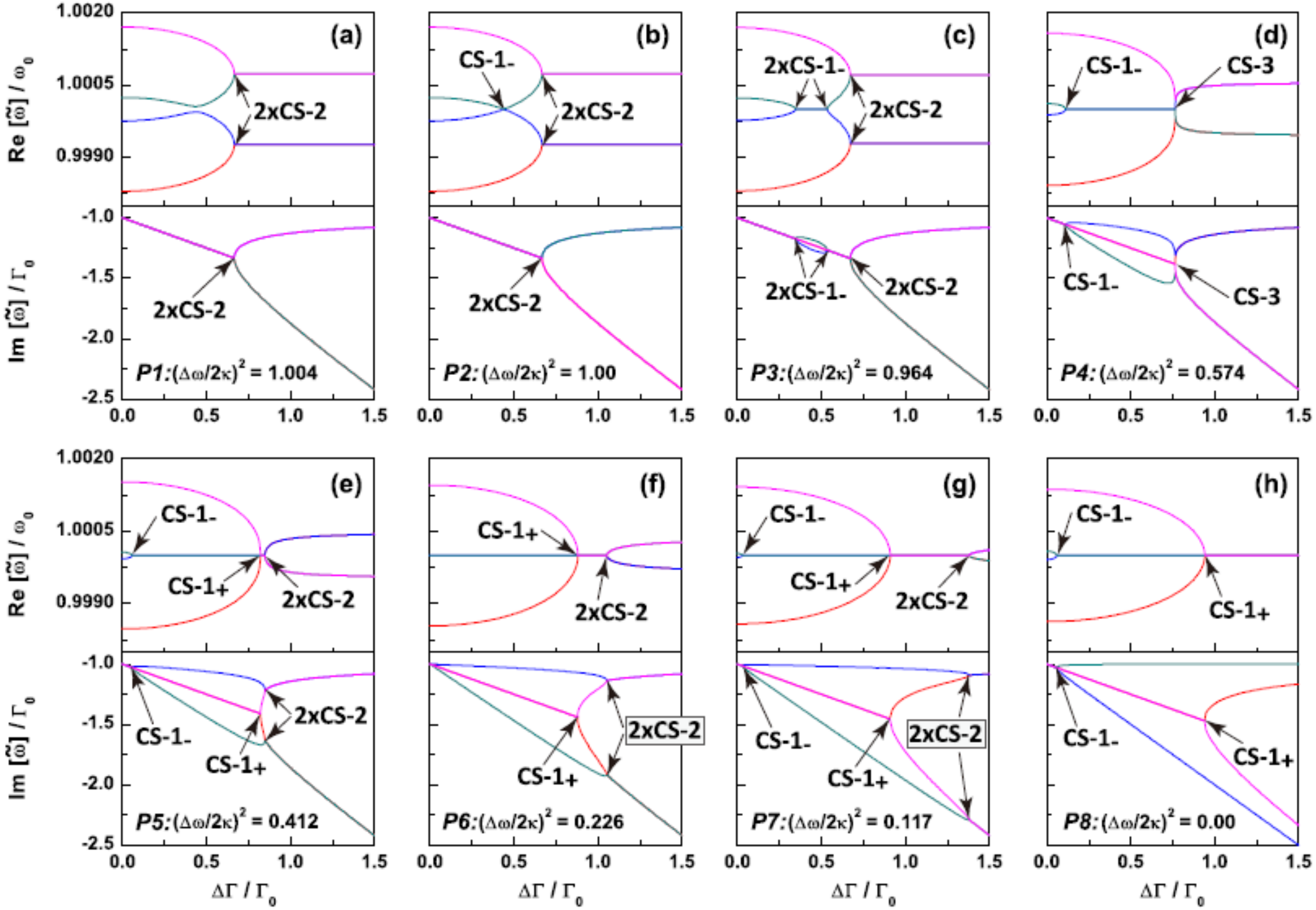
$$\mathrm{CS}-3: \Delta_1 = \Delta_2 = 0$$

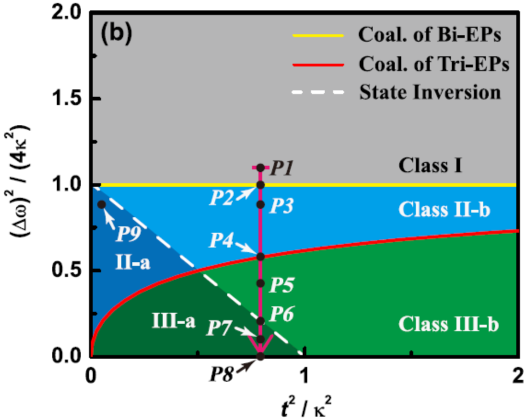
$$(\Delta\omega)^2 = 2\sqrt{t^4 + 4\kappa^2 t^2} - 2t^2$$

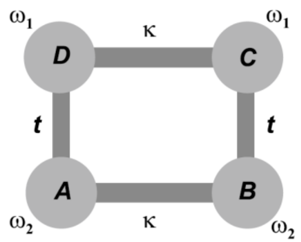
- White dashed line marks the state inversion line : subclasses "a" and "b " with different topological characteristics
- Decrease $(\Delta\omega/2\kappa)^2$ continuously from point P1(0.7744, 1.004) to P8

Eigenfrequency phase diagram

- Exceptional point formation pattern







System parameters:

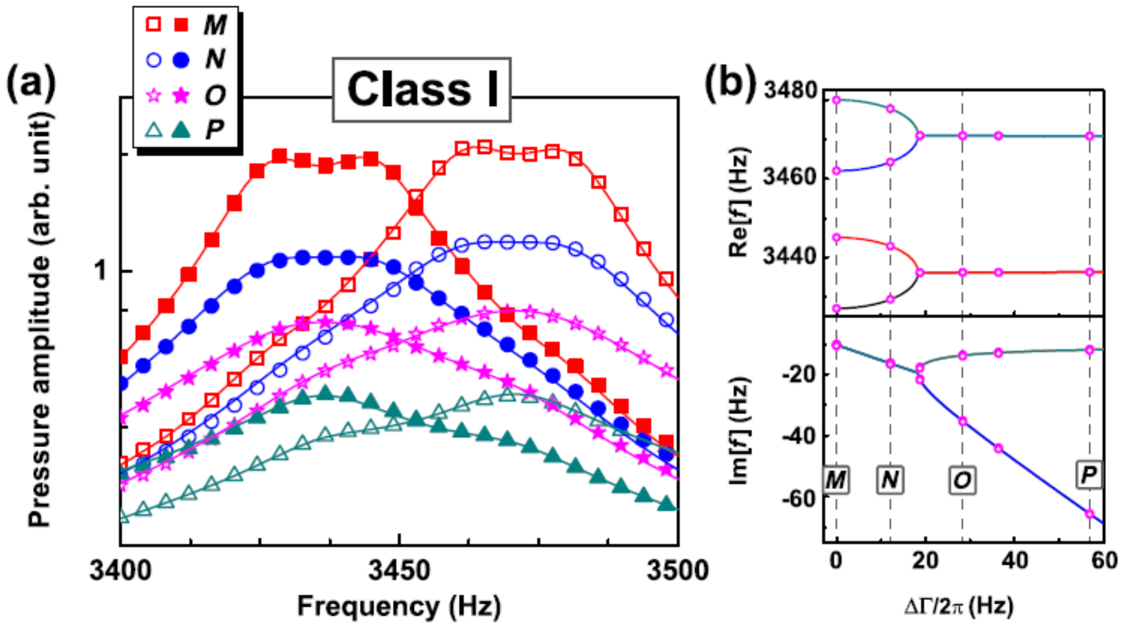
$\omega_0 = 3427.59$ rad/s

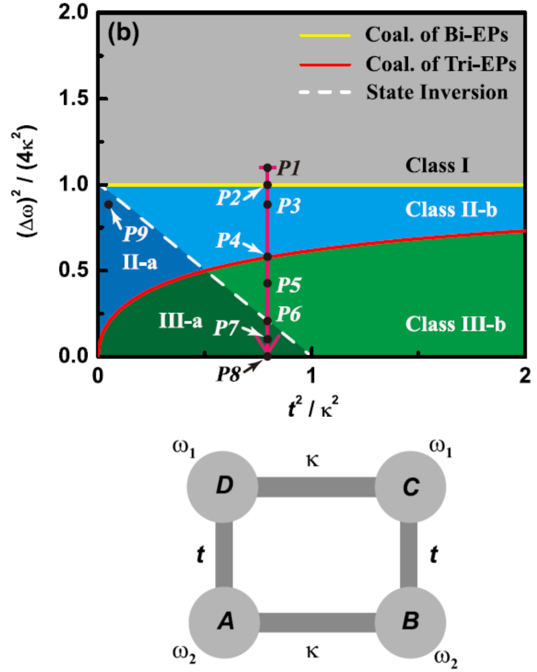
$\kappa = -2.5$ rad/s

$t = -2.2$ rad/s

$\Gamma_0 = 10$ rad/s

Eigenfrequency phase diagram – Class 1



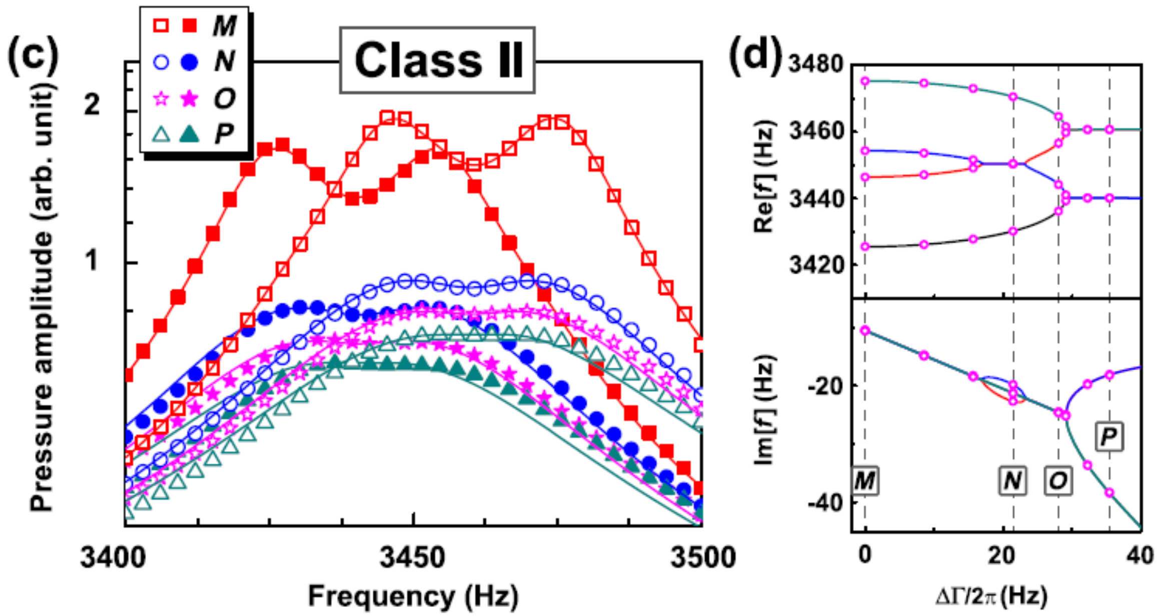


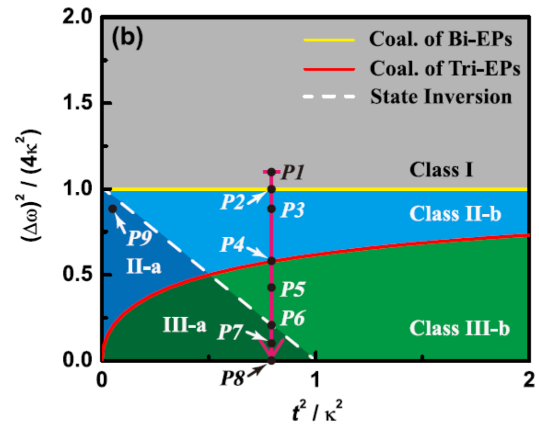
Measured pressure response spectra at cavity B (filled symbols) and cavity D (open symbols)

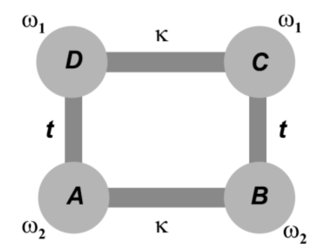
- $(\Delta\omega/2\kappa)^2 = 3.665$, $(t/\kappa)^2 = 0.094$
- Red squares : without loss
- Two EPs are generated at 3436.2 and 3471.0 HZ

A, B : h=50.6 mm, putty 150 mg
C, D : h=50.0 mm
$\kappa \rightarrow$ 1.2 mm tube
$t \rightarrow$ 0.8mm, 0.4mm tubes

Eigenfrequency phase diagram – Class 2





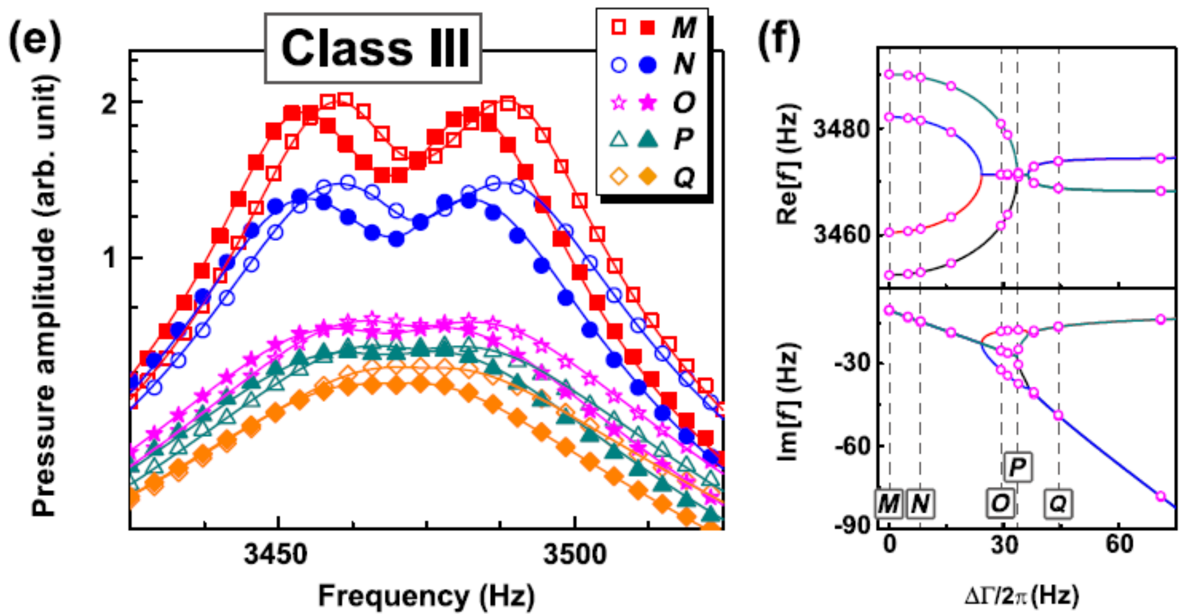


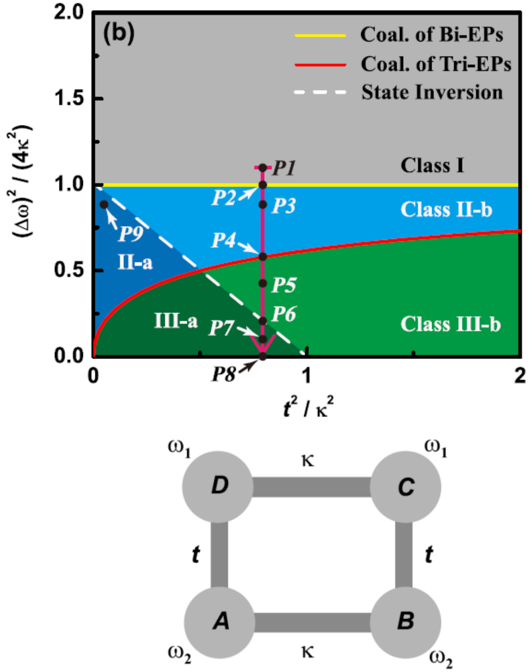
Measured pressure response spectra at cavity B (filled symbols) and cavity D (open symbols)

- $(\Delta\omega/2\kappa)^2 = 0.79$, $(t/\kappa)^2 = 0.017$
- State inversion
- Central coalesced state at 3450 HZ, bifurcate again as $\Delta\Gamma$ increases.
  → This cannot occur in a 2×2 system.

A, B : h=50.6 mm
C, D : h=50.2 mm
κ → 2.0 mm, 0.8 mm tubes
t → 0.4 mm tube

Eigenfrequency phase diagram – Class 3





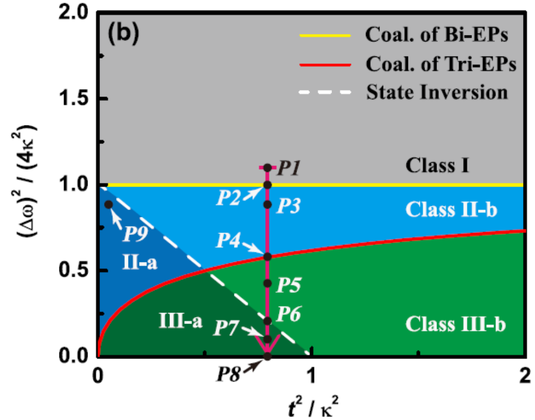
Measured pressure response spectra at cavity B (filled symbols) and cavity D (open symbols)

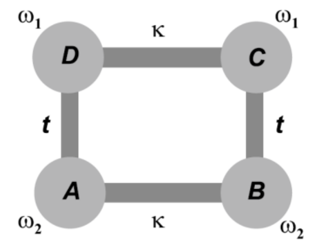
- $(\Delta\omega/2\kappa)^2 = 0.048$, $(t/\kappa)^2 = 0.025$
- A stage with 3 different Re[f]
- A regime with only one real frequency

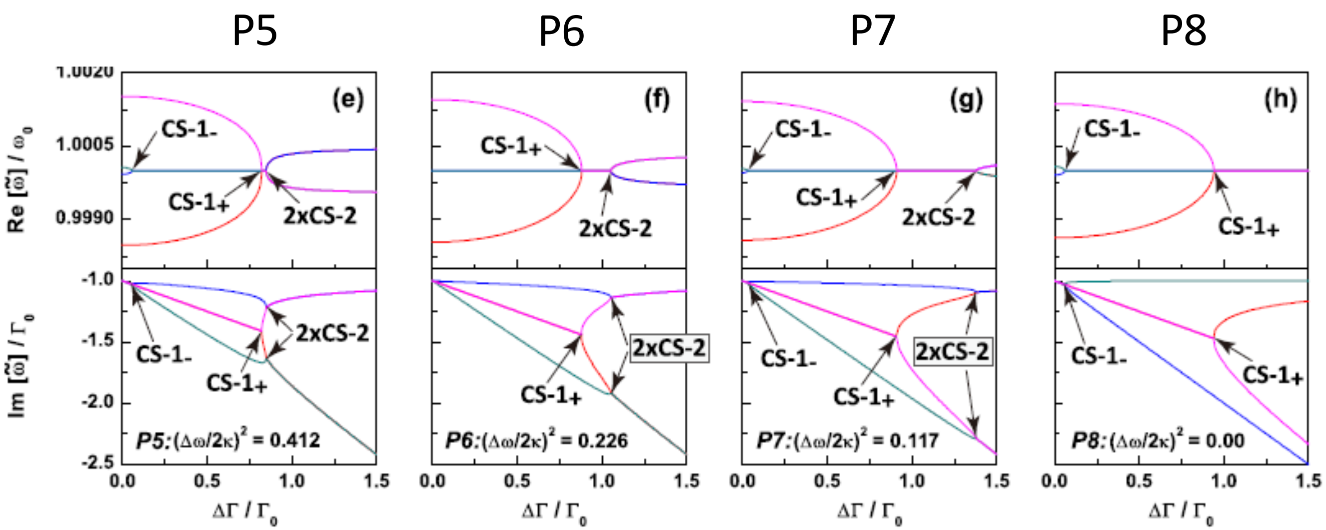
A, B : h=50.2 mm, putty 150 mg
C, D : h=50.0 mm
κ → 2.0 mm, 0.8 mm tubes
t → 0.4 mm, 0.8 mm tubes

Eigenfrequency phase diagram – State inversion line

- At P6, the system configuration meet the boundary of classes 3-a and 3-b
- State inversion line : $4\kappa^2 = 4t^2 + (\Delta\omega)^2$
- Although the EP pattern in (e) and (g) seems to be same, they actually have different chiralities.



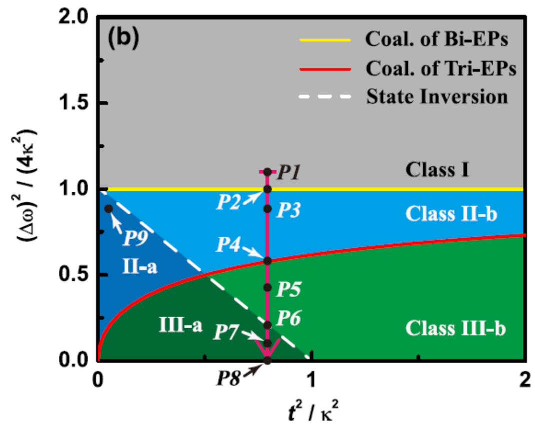


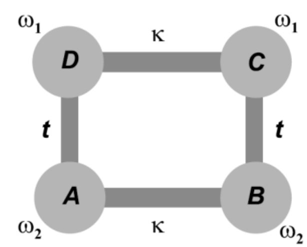


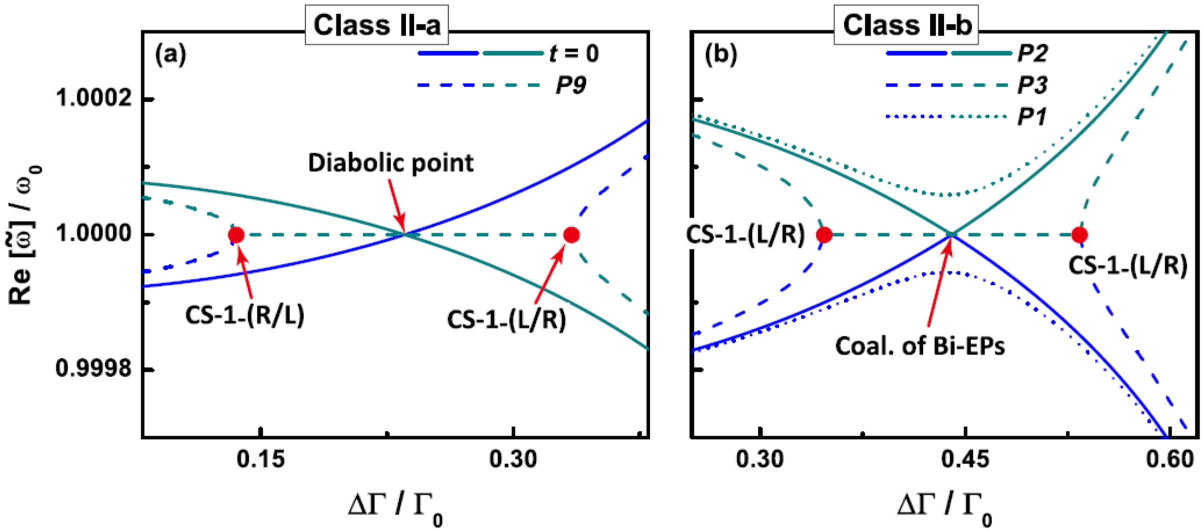
Topological characteristics around singularities

- The chirality of EP

analogous to the polarization of EM waves : $\begin{cases} \dfrac{(-i,1)}{\sqrt{2}} & \text{right} \\ \dfrac{(i,1)}{\sqrt{2}} & \text{left} \end{cases}$







- Class 2-a and 2-b share the same EPFP, but the chirality of EPs is different.
- Class 2-a : two CS-1 have opposite chirality, two EPs cancel each other.
- Class 2-b : two CS-1 have the same chirality, there is a higher-order singularity.

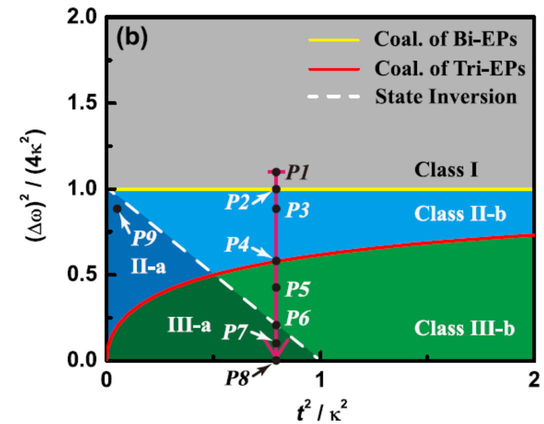
Topological characteristics around singularities

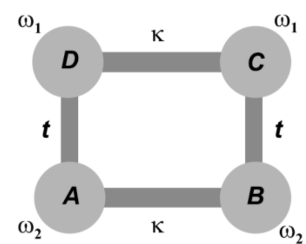
(a) Phase rigidity of all the eigenstates for P2 $r_j = \langle \tilde{\phi}_j^R | \tilde{\phi}_j^R \rangle^{-1}$

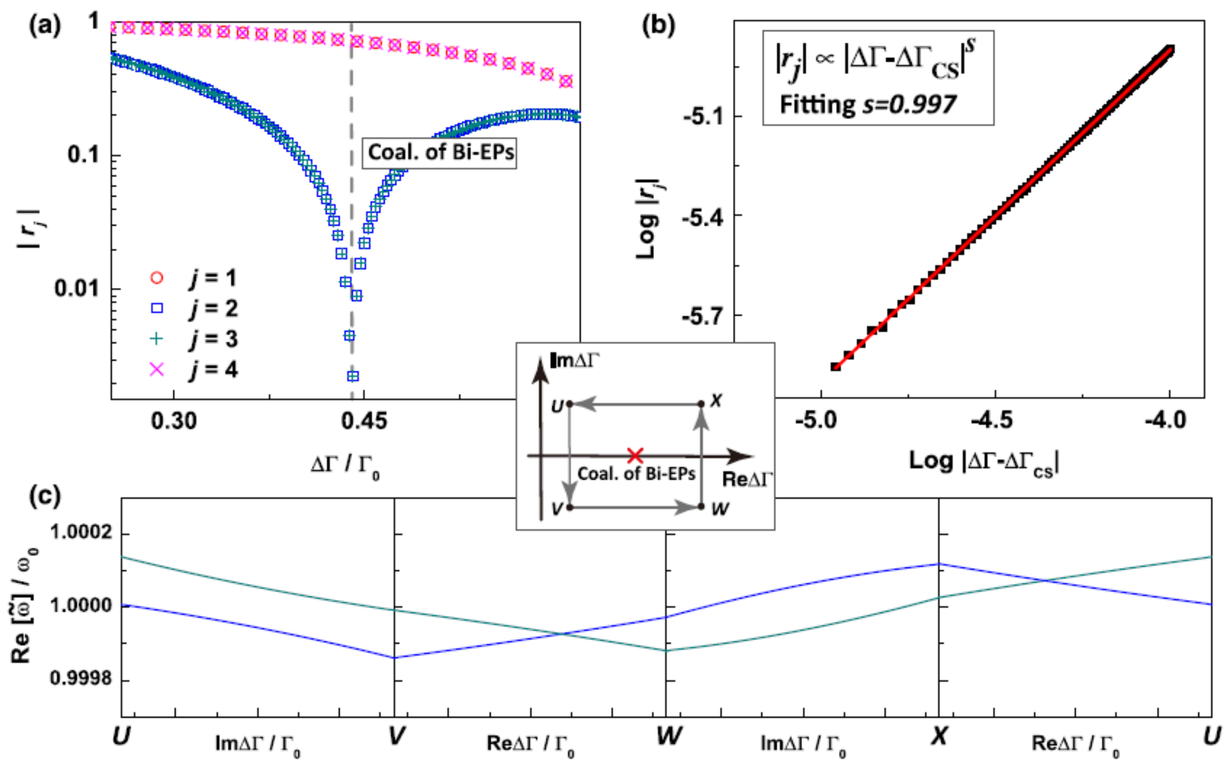
(b) Exponent of 1

(c) Encircling the coalescence point (U→V →W →X →U)

: only 1 cycle is required to bring the two states back to their original positions

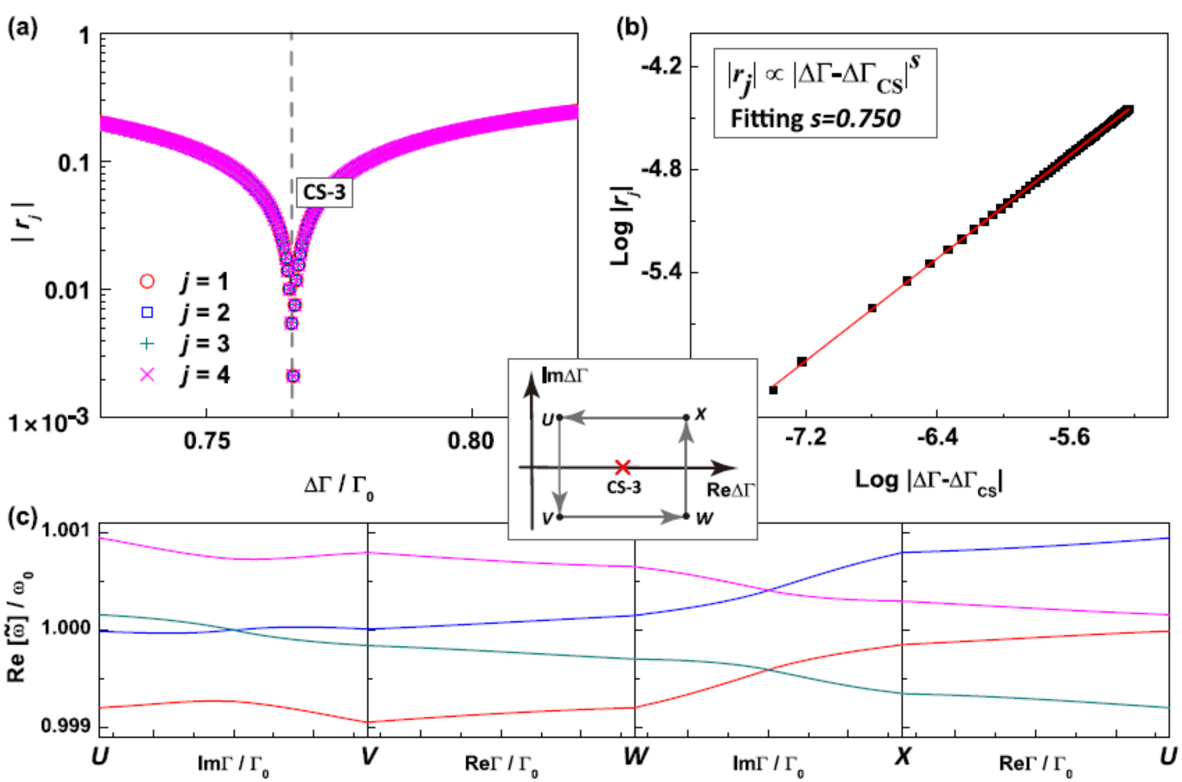


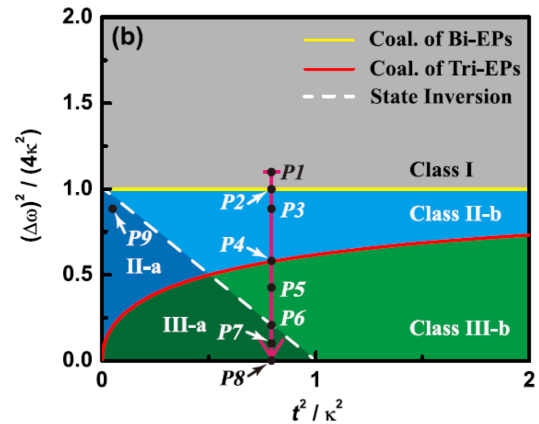


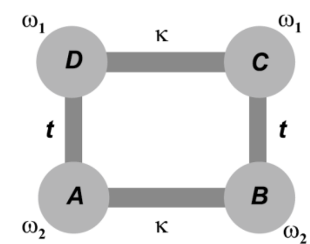


Topological characteristics around singularities

(a) Phase rigidity of all the eigenstates for P4
(b) Exponent of 3/4
(c) Encircling the coalescence point (U→V →W →X →U)
: 4 cycles are required to bring the states back to their original positions







Conclusion

- EP-related physics are expected to be even richer with increased number of connected cavities

: the symmetry of the network and the topology of the connectivity can serve as extra degrees of freedom.

- The new physics obtained here should also apply to EM and matter waves.
- As singularities underlie the essence of EPs, the new singularities found in higher dimensions and their associated topological properties can serve as new platforms for realizing new phenomena.