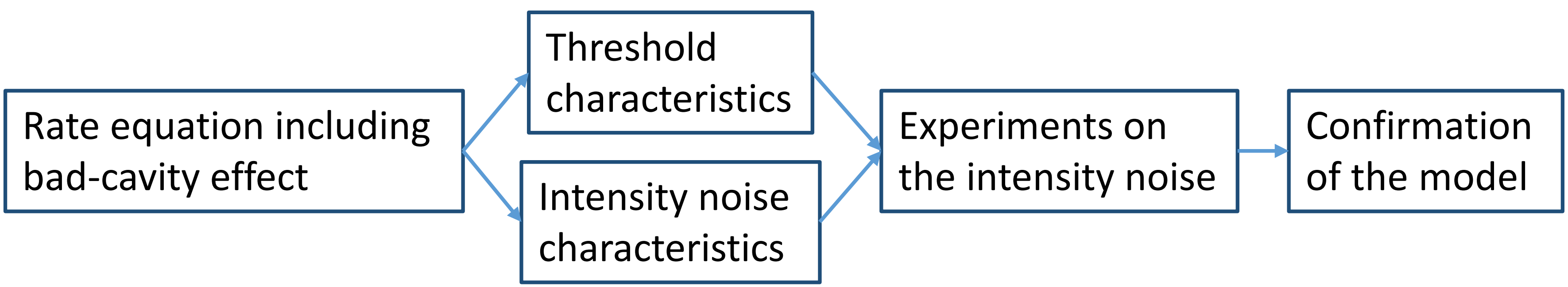
Threshold characteristics and intensity fluctuations of lasers with excess quantum noise

M. A. van Eijkelenborg *et.al.*, Phys. Rev. A **57**, 571 (1998).

Outline

- This paper presents the threshold characteristics and intensity noise of a laser with excess noise.
- A theoretical description of the intensity aspects of excess noise based on laser equations including bad-cavity effects was studied.
- Experimentally, intensity noise and phase noise spectra of HeXe gas lasers operating on either a stable or an unstable cavity had been measured.
- By comparing the measured spectra with the theory, excess noise factor K and the spontaneous-emission factor β were deduced.



Theoretical Model

$$\dot{s} = [G(N) - \boxed{\Gamma_c(N)}]s + R_{sp} + \boxed{f(t)}$$
$$\dot{N} = \Lambda - \gamma_0 N(1 + \beta s)$$

Langevin noise, not important in the threshold characteristics and $(K)\beta \ll 1$ condition needed.

Bad cavity effect,
dressed cavity loss instead of the cold cavity loss Γ_0

s: # of photons in the lasing mode

N: population inversion

$G(N)$: intensity-gain rate

$\Gamma_c(N)$: cavity loss rate of the dressed cavity

R_{sp}: average spontaneous emission rate

$f(t)$: fluctuating term

Λ: pump rate

γ_0: decay rate of inversion

β: fraction of spontaneous emission that ends up in the lasing mode

Assumption: atoms have β^{-1} channels of emission and the lasing mode has K times higher weight than the others.

Threshold characteristics (1/2)

$$\dot{s} = [G(N) - \Gamma_c(N)]s + R_{sp}$$
$$\dot{N} = \Lambda - \gamma_0 N(1 + \beta s)$$

- Using relations below, expand $G(N) - \Gamma_c(N)$ around the threshold and set the time derivative equal to zero.

ad hoc insertion

$$R_{sp} = K\beta\gamma_0 N_2 = KN_{sp}G(N) = KN_{sp}\Gamma_c \quad \textit{where} \quad N_{sp} = N_2/N, G(N) = N\beta\gamma_0$$

$$\Gamma_c(N) = \frac{\Gamma_0}{n_{gr}(N)} = \frac{\Gamma_0}{1 + [\Gamma_0/2\gamma_{gain}][G(N)/G(N_{th})]}$$ (dressed cavity loss rate)

$$\frac{\partial\Gamma_c(N)}{\partial N} = -\frac{\Gamma_c}{2\gamma_{gain}}\frac{\partial G}{\partial N} \approx -\frac{1}{1 + 2\gamma_{gain}/\Gamma_0}\frac{\partial G}{\partial N}$$

$$C = \frac{2 + 2\gamma_{gain}/\Gamma_0}{1 + 2\gamma_{gain}/\Gamma_0}$$ (bad cavity correction)

$$\Lambda_{th} = \gamma_0 N_{th}\,, \Gamma_c = \beta\Lambda_{th}$$

Threshold characteristics (2/2)

$$\dot{s} = [G(N) - \Gamma_c(N)]s + R_{sp}$$
$$\dot{N} = \Lambda - \gamma_0 N(1 + \beta s)$$

- Then we have the equation for s as below:

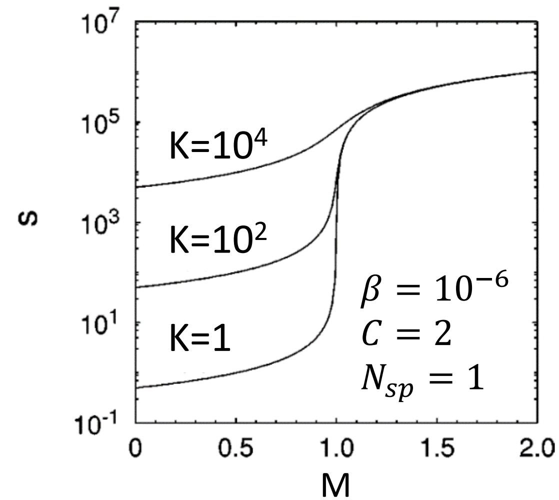
  $C[\Lambda - \Lambda_{th}(1 + \beta s)]s + K\Lambda_{th}N_{sp}(1 + \cancel{\beta s}) = 0\,.$

- By solving the above equation, the solution is:

$$s = \frac{1}{2\beta}\left[(M-1) + \frac{K\beta N_{sp}}{C} + \sqrt{\left(M - 1 + \frac{K\beta N_{sp}}{C}\right)^2 + 4K\beta\frac{N_{sp}}{C}}\right].$$

According to the authors, in the limit $\beta s \ll 1$

$$\boxed{s_0 = \frac{1}{2\beta}\left[(M-1) + \sqrt{(M-1)^2 + 4K\beta\frac{N_{sp}}{C}}\right]}$$



Intensity noise

Fluctuations around the operating point N_0, s_0 (linear approximation):

$$\dot{\sigma} = -\frac{R_{\rm sp}}{s_0}\sigma + \gamma C \beta s_0 \eta + f(t) \qquad (s = s_0 + \sigma)$$

$$\dot{\eta} = -\Gamma_{\rm c}\sigma - \gamma(1+\beta s_0)\eta \qquad (N = N_0 + \eta)$$

Solution: $|\sigma(\omega)|^2 = 4R_{\rm sp}s \Big/ \left| -i\omega + K\frac{N_{\rm sp}\Gamma_{\rm c}}{s_0} + \frac{\gamma C\beta s_0 \Gamma_{\rm c}}{\gamma(1+\beta s_0) - i\omega}\right|^2$

A. Low frequency

$$|\sigma(0)|^2 = \frac{4}{C\beta\Gamma_c} s_0 \Big/ \left(\frac{s_{th}}{s_0} + \frac{s_0}{s_{th}}\right)^2$$

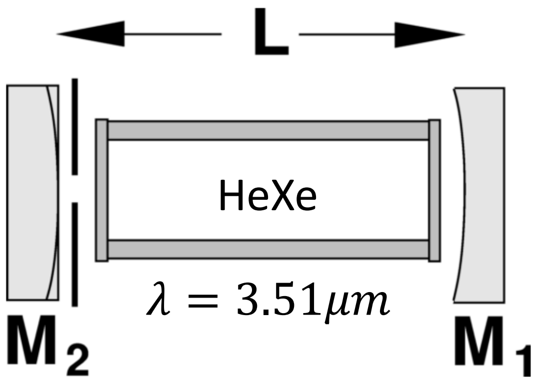
B. Far below threshold

$$|\sigma(\omega)|^2 = \frac{4 s_0^2 \Delta\omega}{\omega^2 + \Delta\omega^2} \quad \mathit{with} \quad \Delta\omega = K\frac{N_{sp}\Gamma_c}{s_0}$$

C. Far above threshold

$$|\sigma(\omega)|^2 = 4R_{sp}s\frac{\gamma_d^2 + \omega^2}{(\omega^2 - \omega_0^2)^2 + \gamma_d^2\omega^2} \quad \mathit{where} \quad \omega_0^2 = \gamma C\beta s_0 \Gamma_c \ and\ \gamma_d = \gamma(1 + \beta s_0)$$

Experimental setup



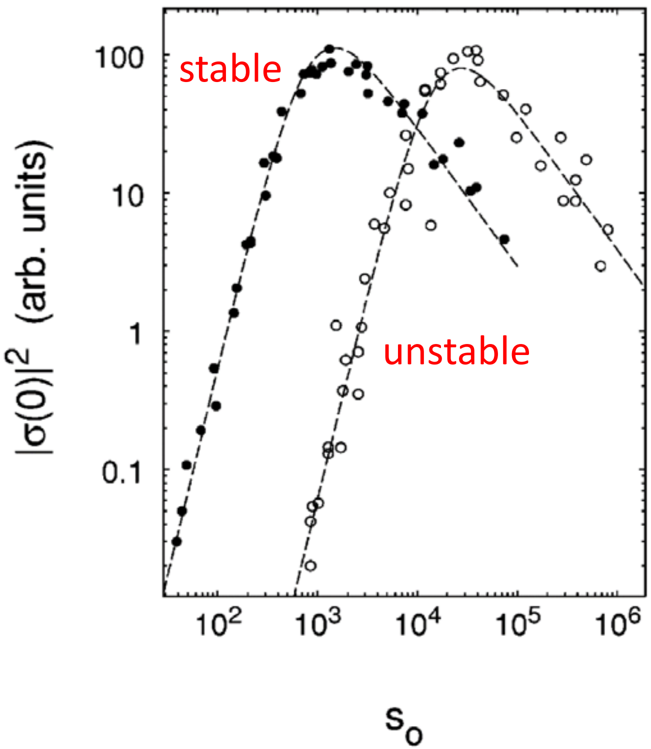
Laser cavity	R_1 (cm)	R_2 (cm)	L (cm)	Γ_c (10^8 s^{-1})	Γ_m (10^8 s^{-1})	C	N_{sp}
stable	−30	∞	9.49	6.84	5.11	1.72	1.4
unstable	−30	+10.4	9.40	8.14	2.67	1.85	1.4–2.2

Determining the dressed-cavity loss rate Γ_c is necessary:

$$\Delta\omega = K\frac{N_{sp}\Gamma_c}{s_0}, \quad |\sigma(0)|^2 = \frac{4}{C\beta\Gamma_c}s_0\Big/\left(\frac{s_{th}}{s_0}+\frac{s_0}{s_{th}}\right)^2$$

- Stable cavity
  Calculated from the mirror reflectivities and transmission of the gain-tube windows.
- Unstable cavity
  Apply an axial magnetic field and measure the frequency pulling (left and right circularly polarization) which depends on the loss rate.

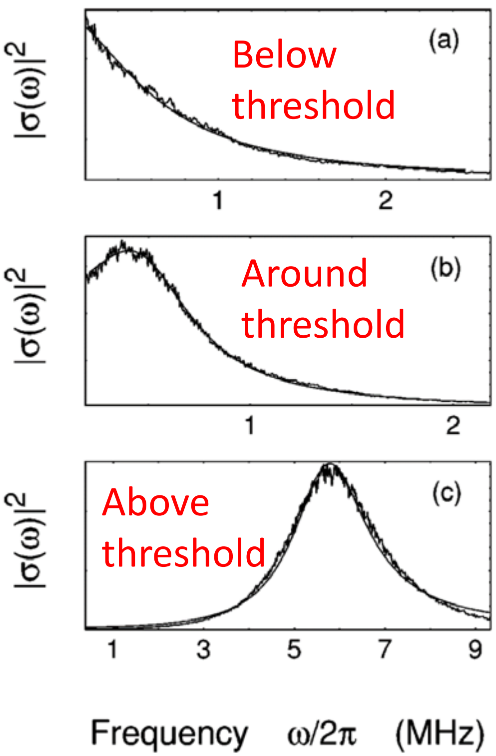
Intensity noise at low frequency

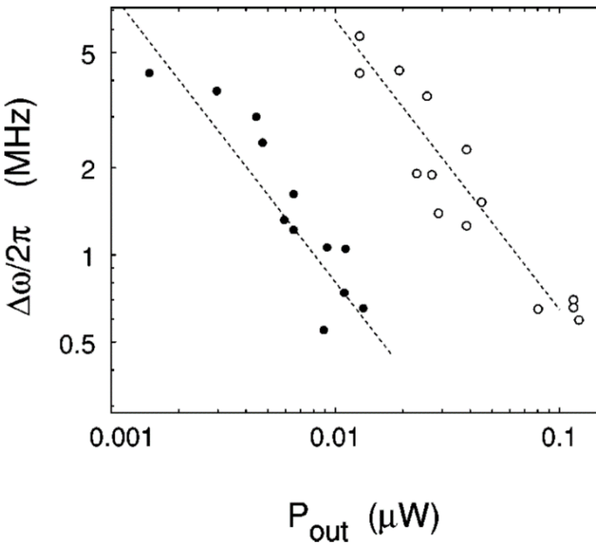


$$|\sigma(0)|^2 = \frac{4}{C\beta\Gamma_c} s_0 \Big/ \left(\frac{s_{th}}{s_0} + \frac{s_0}{s_{th}}\right)^2$$

- Experimental data fits well the theoretical model.
- The fitting provides the value of s_{th} for both cases.
  $s_{th} = 858 \pm 60$ for the stable and
  $s_{th} = (15.9 \pm 1.5) \times 10^3$ for the unstable cavity.
- Using $s_{th}^2 = \frac{KN_{sp}}{C\beta}$,
  $K\beta^{-1} = (9.3 \pm 1.4) \times 10^5$
  for the stable and
  $K\beta^{-1} = (3.4 \pm 0.7) \times 10^8$
  for the unstable cavity.

Intensity noise far below threshold





$$|\sigma(\omega)|^2 = \frac{4s_0^2\Delta\omega}{\omega^2 + \Delta\omega^2}$$

$$\left(\Delta\omega = K\frac{N_{sp}\Gamma_c}{s_0}\right)$$

- Subthreshold Lorentzian spectrum is fitted to data shown in (a).
  $(\Delta\omega/\pi)P_{out} = (8.0 \pm 1.0) \times 10^{-3}\mathrm{HzW}$ for the stable,
  $(\Delta\omega/\pi)P_{out} = (65 \pm 9) \times 10^{-3}\mathrm{HzW}$ for the unstable.
- Using $P_{out} = h\nu\Gamma_m s_0$,
  $K = 1.9 \pm 0.3$ for the stable and
  $K = 24 \pm 4$ for the unstable cavity.

Summary

Laser cavity	K Calculation	K Subthreshold	$K\beta^{-1}$ Low frequency	K Phase noise	β Combining K and $K\beta^{-1}$	β Calculation
Stable	1.1	1.9 ± 0.3	$(9.3\pm1.4)\times10^5$	1.1 ± 0.2	2.0×10^{-6}	3.7×10^{-6}
Unstable	82	24 ± 4	$(3.4\pm0.7)\times10^8$	32 ± 5	0.71×10^{-7}	$(1.2-5.9)\times10^{-7}$

- Influence of excess noise on both the threshold characteristics and the intensity noise of a laser was investigated both theoretically and experimentally.
- Output power at threshold and sub-threshold noise spectra derived, including K.
- Intensity-noise spectra of HeXe lasers studied, both for a stable and an unstable cavity. K and β measured.
- Possibility of thresholdless laser operation by maximizing $K\beta$.